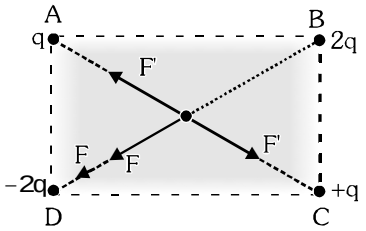
ELECTROSTATICS

EXERCISE –I

1. $[\epsilon_0] = \dfrac{[Q^2]}{[F][4\pi][\sigma^2]} = \dfrac{A^2T^2}{MLT^{-2} \times L^2} = M^{-1}L^{-3}T^4A^2$

2. $\dfrac{K(9e)q}{r^2} = \dfrac{Keq}{(16-r)^2} \Rightarrow \dfrac{3}{r} = \dfrac{1}{16-r} \Rightarrow r = 12$ cm

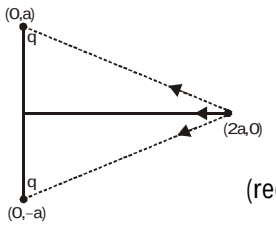
3.



Hence net force is along BD

4. Same charges repels each other.

5.



$F = QE = \dfrac{QKQx}{(R+x^2)^{3/2}}$

$F \propto (-x)$

(required equation for SHM)

7. Force between two line charges On a unit length

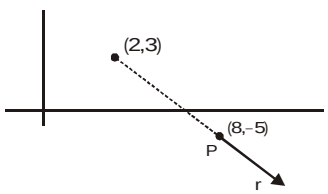
$= \dfrac{2K\lambda}{r} \times \lambda = \dfrac{2 \times 9 \times 10^9 \times (5 \times 10^{-6})^2}{0.1} = 4.5$ N/m

8. $t = \sqrt{\dfrac{2x}{a}} = \sqrt{\dfrac{2x \times m}{qE}} \therefore t \propto \sqrt{\dfrac{m}{q}} \propto \sqrt{m}$

[when x, q & E are same] $\therefore \dfrac{t_2}{t_1} = \sqrt{\dfrac{m_p}{m_e}}$

9. If particle will loose KE (in that direction) against work done by electric field $\dfrac{K}{2} = qEd \Rightarrow E = \dfrac{K}{2qd}$

10.



$E = \dfrac{KQ}{r^2}\hat{r} = \dfrac{9 \times 10^9 \times 50 \times 10^{-6}}{100} \times \left(\dfrac{6\hat{i} - 8\hat{j}}{10}\right)$

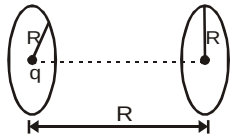
= 4500 V/m.

11. Work done by external force = ΔU

[It is state function]

12. Charge moves $\perp^r$ to the field lines. So the work done will be zero.

13. $U_i = \dfrac{KQ_1q}{R} + \dfrac{KQ_2q}{\sqrt{R^2+R^2}}$



$U_f = \dfrac{KQ_1q}{\sqrt{R^2+R^2}} + \dfrac{KQ_2q}{R}$

Work done by external force : $\Delta U = U_f - U_i$

14. By mechanical energy conservation

$(PE + KE)_i = (PE + KE)_f$

$0 + \dfrac{1}{2}mv^2 + 0 = \dfrac{KQ^2}{d} + \dfrac{1}{2}m\left(\dfrac{v}{2}\right)^2 x2$

($\because$ from momentum conservation at closet approach, both particle will move with a common speed v/2)

$\therefore d = \dfrac{4KQ^2}{mv^2}$

15. $V = (n-1)\dfrac{KQ}{r}$, $E = \dfrac{KQ}{r^2} \Rightarrow \dfrac{V}{E} = (n-1)r$

16. Let distance of closest approach be 'd' then

$\dfrac{1}{2}mv^2 = \dfrac{K(2e)(9)(2e)}{d^2} \Rightarrow d = 10^{-12}$ cm

17. F = QE $\Rightarrow$ 300 = 3 x E $\Rightarrow$ E = 100 N/C

$E = \dfrac{dV}{dx} \Rightarrow \Delta V = 100 \times \dfrac{1}{10} = 10$ V

18. Slope from x axis

y = 3 + x; $\tan\theta = 1 \Rightarrow \theta = 45°$

$\therefore$ Electric field in vector from $\vec{E} = 100\left(\dfrac{\hat{i}+\hat{j}}{\sqrt{2}}\right)$

$\therefore V = -\displaystyle\int_{(3,1)}^{(1,3)} \vec{E} \cdot d\vec{r}$

$V = = \dfrac{-100}{\sqrt{2}}\left[\displaystyle\int_3^1 dx + \int_1^3 dy\right] = 50\sqrt{2}\,[-2+2] = 0$

Alternate solution : The direction of electric field and the slope of line A (3,1) & (1,3) is $\perp$ to each others so the dot product $\vec{E}.d\vec{r}$ becomes zero.

19. Equipotential lines are always $\perp^r$ to the electric field strength lines .
$\because$ slope of equipotential lines =2
$\therefore$ Slope of electric field must be = – 1/2
$\Rightarrow$ Electric field strength vector $= -8\hat{i} + 4\hat{j}$

20. $E = -\left(\frac{\partial V}{\partial x}\hat{i} + \frac{\partial V}{\partial y}\hat{j} + \frac{\partial V}{\partial z}\hat{k}\right) = -k\left[4x\hat{i} - 2y\hat{j} + 2z\hat{k}\right]$

$= -k\left[\sqrt{16+4+4}\right] = 2k\sqrt{6}$

21. Slope of equipotential lines will be = 1/2
$\therefore$ Slope of electric lines must be = – 2

OR

$$E_x = -\left(\frac{\Delta V}{\Delta x}\right)_{y=constant} = -\frac{(4-2)V}{(4-2)cm} = -100\ V/m$$

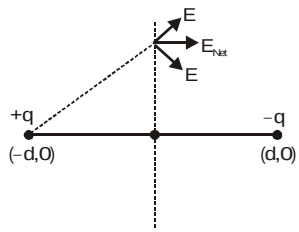
$$E_y = -\left(\frac{\Delta V}{\Delta y}\right)_{x=constant} = -\frac{(2-4)V}{(1-0)cm} = 200\ V/m$$

22. $-\int_{\infty}^{\ell=0} \vec{E}.\overrightarrow{d\ell}$ is the potential at the centre of the ring which is

$$V = \int dV = \int \frac{Kdq}{0.5} = \frac{Kq}{0.5}$$

$$V = \frac{9\times10^9 \times 1.11\times10^{-10}\times 2}{1} = 2 \text{ volt}$$

23. In figure (–d,0) to (d,0) on x–axis the direction of $\vec{E}$ in +ve x–axis and left side of (–d,0) the direction in –ve x–axis, but on y–axis, at any point the net electric field along the x–axis.



24. W=(–pEcosθ) – (–pE)=PE(1–cosθ)

25. Electric field lines can't be closed

26. From flux = $\int \vec{E}.\overrightarrow{ds} = \frac{Eq_{enclosed}}{\epsilon_0}$ $\therefore$ flux = $\frac{Q}{\epsilon_0}$

27. $q = \epsilon_0 \int \vec{E}_x.d\vec{x} = \epsilon_0 \times 600\int_{0.1}^{0.2}\frac{dx}{\sqrt{x}} = 7\times10^{-12}C$

28. Area(100 m²) in xy plane so area vector in $\hat{k}$ so flux)

$= \int \vec{E}_z \cdot d\vec{S} = \sqrt{3}\hat{k}.100\hat{k} = 173.2$

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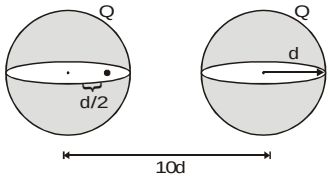
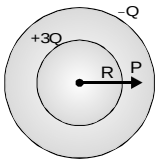
29.



$$F = \frac{1}{4\pi\epsilon_0}\times\frac{Qq}{\left(\frac{19}{2}d\right)^2} + 0 = \frac{qQ}{361\pi\epsilon_0 d^2}$$

30.



$$E_P = \frac{3Q}{4\pi\epsilon_0 R^2}$$

31. In hollow sphere potential remains constant inside the shell.

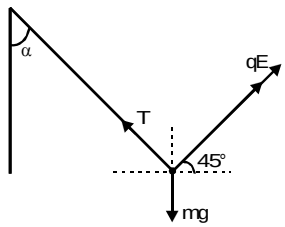
32. The potential difference (work done against field) between the shell & sphere is due to the field present inside that region which is only due to the sphere.

33. For any metallic surface, electric field lines are $\perp^r$ to the surface.

34. 1, 2, & 3 are wrong because in metallic solid sphere, there is no field inside the sphere. Option (4) is correct from given reason and also it has field lines perpendicular to the surface (as required in metallic surface)

EXERCISE –II

1. T sinα = qE cos 45° and Tcosα+qE sin 45° = mg

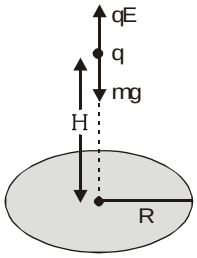


2. $mg = q \times \frac{KQH}{(R^2+H^2)^{3/2}}$

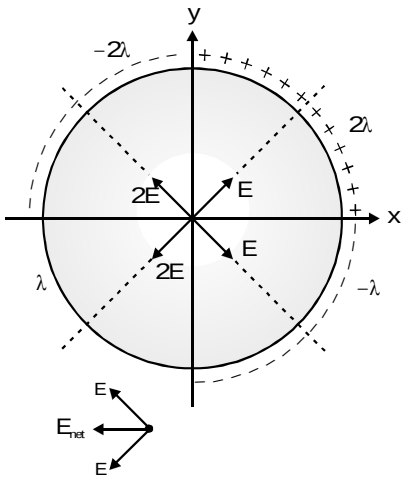
The field due to ring on its axis will be maximum at

$H = \pm \frac{R}{\sqrt{2}}$ i.e. above that point qE force will decrease and resultant force becomes in downward direction (equilibrium position) and also in same way for below point.

$$mg = q \times \frac{KQH}{(R^2 + H^2)^{3/2}}$$



3. $E_{net} = \sqrt{2}E = \sqrt{2}\left[\frac{\lambda\sqrt{2}}{4\pi \epsilon_0 R}\right] \Rightarrow \vec{E} = \frac{\lambda(-\hat{i})}{2\pi \epsilon_0 R}$

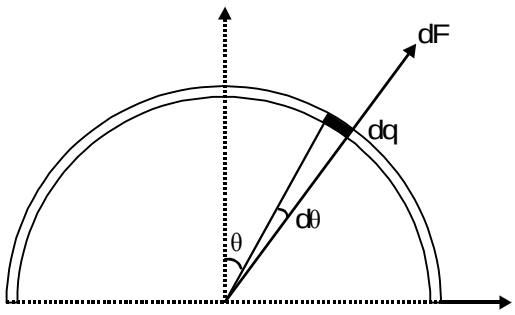


4. Potential at any point is a scalar quantity.
∵Positive charge = negative charge i.e. the net charge is equal to zero. Hence potential is zero. Electric field is a vector quantity so it depends not only distance but also the way of distribution of charge.

5. $E = \frac{KQr}{(R^2 + r^2)^{3/2}}$.

It is maximum at $r = \pm \frac{R}{\sqrt{2}}$ and also E is not a linear function of r.

6.



$$dF = (dq)\left(\frac{\lambda}{2\pi \epsilon_0 R}\right) = \left(\frac{qd\theta}{\pi}\right)\left(\frac{\lambda}{2\pi \epsilon_0 R}\right)$$

$$F_{net} = \int_0^{\pi/2} 2dF\cos\theta = \frac{q\lambda}{\pi^2 \epsilon_0 R}$$

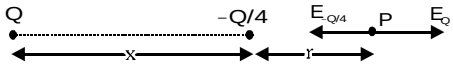
$$\Rightarrow \int_0^{\pi/2} \cos\theta d\theta = \frac{q\lambda}{\pi^2 \epsilon_0 R}$$

7. At point A the field lines are much closer than B hence $E_A > E_B$.
Work done by external force in direction of field lines is negative, hence $V_B > V_A$.

8. $V = \frac{KQ}{r} = \frac{9 \times 10^9 \times 10^{-8}}{\sqrt{(1-4)^2 + (3-7)^2 + (2-2)^2}} = 18V$

and electric field lines will be in all three direction.

9. No point exist in between the charges where field is zero.



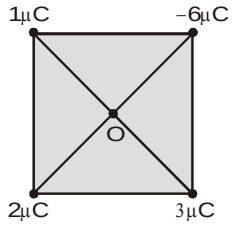
$$E = \frac{KQ}{(r+x)^2} - \frac{K\frac{Q}{4}}{r^2} = 0 \Rightarrow r = x$$

10. If Y is fixed i.e. another force is exist on Y additional to the mutual interaction between X & Y. So the net force on system (X &Y) is not zero so p is changed but total energy always remain conserved.

11. Equal electrostatic force (mutual interaction) acts on X and Y but in opposite direction which accelerate Y but retards X. After long time the velocity of X becomes zero while Y becomes u.

12. Here potential decreases 2V as we moves unit distance. Hence for point (1, 1, 1) from (0, 0, 0). The total potential decrease is 2 +2 +2 = 6V. Hence the potential at point will (10–6) = 4V.

13. Here total charge is zero.



Any point on z–axis, having the same distance from the each vertex of a square. So the potential due to all is zero at that point.

14. V_A = 3 volt, V_B = 7 volt
From energy conservation $(\text{Energy})_A = (\text{Energy})_B$
$\Rightarrow 1/2\ mv^2 - e \times 7V = -e \times 3V + 0$
$\Rightarrow 1/2\ mv^2 = 4eV$

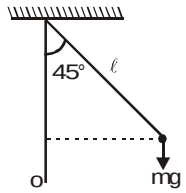
15. Velocity due to acceleration

$$= \frac{10^{-6} \times 300}{10^{-3}} \times 10 = 3\ m/s$$

Then the resultant velocity may be between 1 m/s and 7 m/s

16. If electric field and gravitational field will be in same line then the path may be straight line otherwise parabola.

17. By work energy theorem $W_g + W_{elec} = \Delta KE$

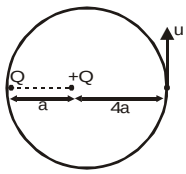


$$mg\ell\left(1 - \frac{1}{\sqrt{2}}\right) + qE \times \frac{\ell}{\sqrt{2}} = \frac{1}{2}mv^2 - 0$$

$$\left(\because E = \frac{mg}{q}\right) \quad u = \sqrt{2g\ell}\ ;\ W = \frac{u}{\ell} = \sqrt{\frac{2g}{\ell}}$$

18. Given V_A due to +Q is V, so V_B = 4V

By energy conservation



$$\frac{1}{2}mu^2 + qV = 4qV$$

$$\Rightarrow u = \sqrt{\frac{6qV}{m}}$$

19. As $E = \dfrac{\lambda}{2\pi \epsilon_0 r}$

so $V = \dfrac{\lambda}{2\pi \epsilon_0}\ell_n r \Rightarrow W = \dfrac{\lambda}{2\pi \epsilon_0}\ell n\left|\dfrac{r_2}{r_1}\right|$

$$W = \frac{\lambda}{2\pi \epsilon_0}\ell n\left(\frac{\sqrt{2}}{\sqrt{1}}\right) + \frac{2\lambda}{2\pi \epsilon_0}\ell n\left(\frac{\sqrt{2}}{\sqrt{2}}\right) + \frac{3\lambda}{2\pi \epsilon_0}\ell n\left(\frac{\sqrt{2}}{\sqrt{1}}\right)$$

$$= \frac{\lambda}{2\pi \epsilon_0}\ell n2$$

20. $E = -\left(\dfrac{\partial V}{\partial x}\hat{i} + \dfrac{\partial V}{\partial x}\hat{j} + \dfrac{\partial V}{\partial z}\hat{k}\right)$

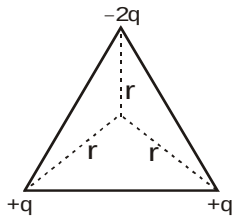
Here $\dfrac{\partial V}{\partial x} = -\dfrac{V}{X_0}$ $\therefore E_{min} = \dfrac{V}{x_0}\hat{i}$.

21. The mass of deuteron is twice,so momentum is different.

22. Potential at centroid of Δ

$$= \frac{Kq}{r} + \frac{Kq}{r} - \frac{K2q}{r} = 0$$

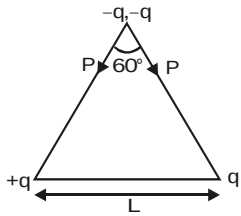
Dipole moment of system



$= 2p \cos 30° = \sqrt{3}p$

$= \sqrt{3} \times q \times L$

$(\because p = q \times L)$



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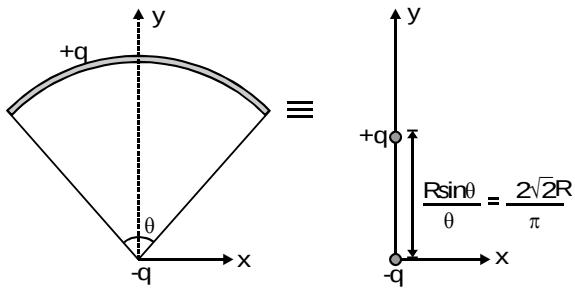


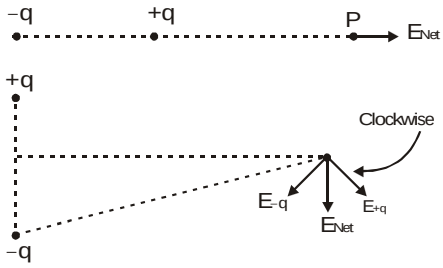
Dipole moment

$$= (q)\left(\frac{2\sqrt{2}R}{\pi}\right) = \frac{2\sqrt{2}qR}{\pi}$$

24. Force on a dipole $F = p\dfrac{\partial E}{\partial x}$

At $x = \dfrac{R}{\sqrt{2}}$ $\dfrac{\partial E}{\partial x} = 0$, so F = 0

25.



26. Interaction energy $U = -\vec{p}_1 \cdot \vec{E}$

where $\vec{E} = \frac{2Kp_2 \cos\theta}{r^3}\hat{r} + \frac{Kp_2 \sin\theta}{r^3}\hat{\theta}$

and $\vec{p}_1 = p_1\hat{r}$

Therefore $U = -\frac{2Kp_1p_2 \cos\theta}{r^3}$

27. Electric flux depends on the total charge enclosed by the closed surface S. Hence flux is related with charge Q_1. Electric field at a point is the vector sum of the all electric field intensities due to all charges.

28. $\because$ Electric flux depends only on the total charge enclosed by the Gaussian surface. Hence[A].

Potential at a distance r = $\frac{Kq}{r}$

Total charge of dipole = 0

29. $\because$Electric field

$= \frac{\sigma}{\epsilon_0}$ due to infinitely uniform charged.

Electric flux

$= \frac{\Sigma q_{enclosed}}{\epsilon_0} = \int \frac{\sigma}{\epsilon_0} \times [\pi R^2 - \pi x^2]$

30. Potential at 5 cm from surface = $\frac{KQ}{R+5} = 100$

Potential at 10 cm from surface

$= \frac{KQ}{R+10} = 75 \Rightarrow$ R = 10 cm

$\therefore$ Potential at surface $= \frac{KQ}{R} = \frac{100 \times 15}{10} = 150V$

Electric field on surface= $\frac{KQ}{R^2} = \frac{100 \times 15V \times cm}{100cm^2}$

= 1500 V/m

31. As the electric field converges at the origin so total charge contained in any spherical volume, irrespective of the location, is negative.

By Gauss theorem $\int \vec{E} \cdot d\vec{s} = \frac{q}{\epsilon_0}$

We have $-E(4\pi r^2) = \frac{q}{\epsilon_0} \Rightarrow q = -3 \times 10^{-13}$ C

32. Energy at surface = Energy at centre

$\frac{1}{2}mu^2 + \frac{Kq \times q}{R} = \frac{3}{2}\frac{Kq}{R} \times q + 0$

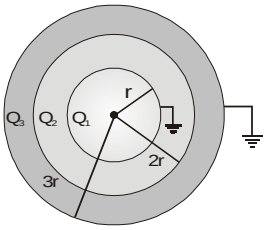
$\therefore$ U= $\frac{q}{\sqrt{4\pi \epsilon_0 mR}}$

33. $\frac{KQ}{R} + \frac{1}{2}mu^2 = \frac{KQ}{2R^3}\left(3R^2 - \frac{R^2}{4}\right) + 0$

$\frac{1}{4\pi \epsilon_0 R} \times \rho \times \frac{4}{3}\pi R^3 + \frac{1}{2}mu^2 = \frac{11 \times \rho \times \frac{4}{3}\pi R^3}{8 \times 2R^2}$

$\therefore u = \left[\frac{\rho R^2}{4m \epsilon_0}\right]^{1/2}$

34.



Potential at outermost shell

$= \frac{KQ_1}{3r} + \frac{KQ_2}{3r} + \frac{KQ_3}{3r} = 0$

$\Rightarrow Q_1 + Q_2 + Q_3 = 0$ ...(i)

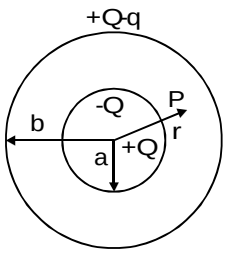
and potential at innermost surface

$\frac{KQ_1}{r} + \frac{KQ_2}{2r} + \frac{KQ_3}{3r} = 0$

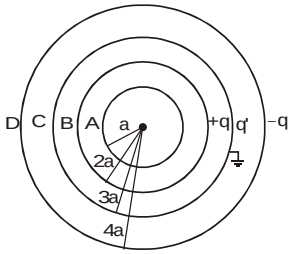
$\Rightarrow \frac{K[6Q_1 + 3Q_2 + 2Q_3]}{6r} = 0$ ....(ii)

From eq. (i) & (ii) $Q_1 = -\frac{Q_2}{4}$ and also $\frac{Q_3}{Q_1} = 3$.

35. Due to the induction, the opposite nature of charge is induced at near by surface.

36. 

from figure $V_P = \frac{K(Q-q)}{b}$

37. 

Potential at C = $\frac{Kq}{3a} - \frac{Kq}{4a} + \frac{Kq'}{3a} = 0 \Rightarrow q' = -\frac{q}{4}$

Potential at A = $\frac{Kq}{2a} - \frac{Kq}{4a} - \frac{K\frac{q}{4}}{3a} = \frac{Kq}{6a}$.

Hence $V_A - V_C = \frac{Kq}{6a}$

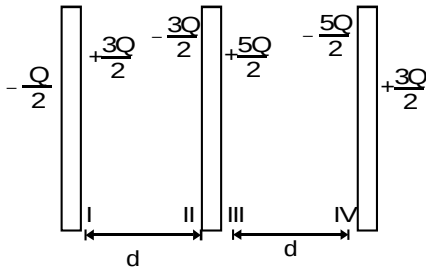
39. Surface charge density at inner surface of X is

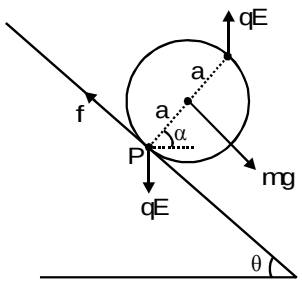
$\sigma = \frac{Q}{2A}$

$\therefore$ Electric field at B due to this is $\frac{\sigma}{2\epsilon_0} = \frac{Q}{4\pi\epsilon_0}$

Towards right and in same direction of same value due to induced charged present inside surface of plate Y.

40. Final charge distribution on plate



41. 

For rotational equilibrium, $\sum \tau_P = 0$

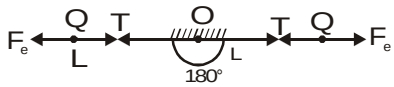
$\Rightarrow$ mgacosα –qE (2acosα) = 0 $\Rightarrow$ $E = \frac{mg}{2q}$

EXERCISE –III

FILL IN THE BLANKS

2. Magnitude of electric field is greatest at a point where electric lines of force are most close to each other.

3. Due to electrostatic repulsion the charges will move as farthest as possible and the angle between the two strings will be 180° as shown in figure. Tension in each string will be equal to the electrostatic repulsion between the two charges. Thus,



$T = F_e = \frac{1}{4\pi\epsilon_0}\frac{Q \times Q}{(2L)^2} = \frac{Q^2}{16\pi\epsilon_0 L^2}$

4. $W_{Fe} = \vec{F}.\vec{d} = (qE\hat{i})\cdot[\vec{r}_s - \vec{r}_p]$

$= qE\hat{i}\cdot(-a\hat{i} - b\hat{j}) = -qEa$

5. $\vec{E} = \left[\frac{\partial V}{\partial x}\hat{i} + \frac{\partial V}{\partial y}\hat{j} + \frac{\partial V}{\partial z}\hat{k}\right]$; where $V = 4x^2$.

Therefore $\frac{\partial V}{\partial x} = 8x$ and $\frac{\partial V}{\partial y} = 0 = \frac{\partial V}{\partial z}$

Therefore $\vec{E} = -8x\hat{i}$

At (1m, 0, 2m) $\vec{E}$ is $-8\hat{i}$ V/m

6. Force on –q due to charges at 1 & 4 are equal and opposite. Similarly, forces on –q due to charges at 2 and 5 are also equal and opposite. Therefore, net force on –q due to charge at 1, 2, 4 and 5 is zero. Only unbalanced force is between –q and +q at 3 which is equal to $\frac{1}{4\pi\varepsilon_0}\cdot\frac{q^2}{L^2}$ = $9.0 \times 10^9 \frac{q^2}{L^2}$ (attraction).

MATCH THE COLUMN

1. Electric field due to metallic plates remains same and constant at near by points.

[A] For $\sigma_1 + \sigma_2 = 0 \Rightarrow \sigma_1 = -\sigma_2$
$\therefore$ Electric field at a point is equal & opposite in direction.

[B] $\sigma_1 + \sigma_2 > 0 \Rightarrow \sigma_1 \& \sigma_2$ [densities] either both positive or opposite but positive has a greater magnitude. So the net electric field will be away from the plates in region I & III.

[C] Same explanation according to [B].

2. Electric field $\vec{E} = -\frac{\partial V}{\partial x}\hat{i} - \frac{\partial V}{\partial y}\hat{j}$

For (A) : $\vec{E} = -\hat{i} + \frac{1}{\sqrt{3}}\hat{j}$

For (B) : $\vec{E} = +\hat{i}$

For (C) : $\vec{E} = -\hat{i} + \sqrt{3}\hat{j}$

For (D) : $\vec{E} = +\frac{1}{\sqrt{3}}\hat{i} + \sqrt{3}\hat{j}$

3. $\because$ Electric field due to an electric dipole at a point on equatorial line of dipole makes either 0° or 180° with the dipole moment of another dipole.

$\therefore$ Torque on dipole $\vec{\tau} = \vec{P} \times \vec{E}$ becomes zero ($\because$ θ = 0 or θ = 180°) Hence in column II, [P] option is suit for every queries for column I.
Electrostatic potential energy (U) =– PE cosθ
θ = Angle between moment & electric field.
[A] Here θ = 180° $\therefore$ U = – PEcos 180° = PE (+ve)
[B] Here θ=0° $\therefore$ U = –PE cos θ = –PE (–ve)
[C] & [D] : θ < 90° $\therefore$ U = –ve

4. (A) Electric field at a point is the vector sum of all individual fields at that point

(B) Electric flux $\oint \vec{E} \cdot d\vec{S} = \frac{q_{enc}}{\epsilon_0}$

(C) Electric flux $\oint \vec{E} \cdot d\vec{S} = \frac{q_{enclosed}}{\epsilon_0}$

5. (A) Initially, the potential difference exist between both shells, so positive charge is flow from high to low potential.
Every system wants to acquire minimum potential energy if possible for stability. So charge flown to achieve it.

(B) As explained in [A], charge flow does not depend on the size of sphere.

(C) Charge flow through wire until the potential becomes same for both shells.

(D) Potential is same everywhere inside a conducting shell. So no charge is flow through connecting wire, so no heat is produced.

7. Electrostatic potential energy
= self energy + interaction energy

(A) Self energy of uniformly charged thin shell

$= \frac{KQ^2}{2a}$ (radius a)

(B) Self energy $(= \frac{KQ^2}{2 \times \frac{5a}{2}}$ &

Interaction energy $= \frac{KQ^2}{\frac{5a}{2}}$

(C) Self energy of solid sphere of radius 'a'= $\frac{3KQ^2}{5a}$

COMPREHENSION BASED QUESTIONS

Comprehention# 1

1. Velocity of B, when it strikes 'A' is

$$v_B = \sqrt{2 \times \frac{1 \times 10}{1} \times 1.8} = 6 \text{ m/s}$$

From COLM between 'A' & 'B' is

0 + 1 × 6 = (1+1) × v = 3m/s [left]

2. Equilibrium position $\Rightarrow \Sigma\vec{F}_{ext} = 0$

Kx = 10 $\Rightarrow$ x = $\frac{10}{18} = \frac{5}{9}$ (Left from x = 0)

3. At equilibrium position the spring is compressed by x = 5/9. Let the amplitude of oscillation be 'A'

$$\therefore \frac{1}{2}KA^2 = \frac{1}{2}Kx^2 + \frac{1}{2}mv^2$$

$$A = \sqrt{x^2 + \frac{m}{K}v^2} = \sqrt{\frac{25}{81} + \frac{2}{18} \times 9} = \frac{\sqrt{106}}{9}$$

Comprehention# 2

1. $\because$ Gravity is absent and path of the particle is parabola i.e. a downward (qE) force is necessary.

2. Acceleration of the particle is given by

$$a = \frac{qE}{m} \Rightarrow a \propto \frac{q}{m}$$

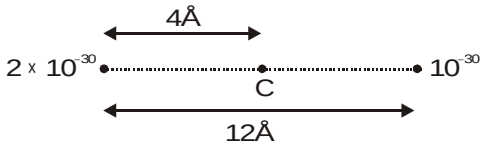
3. $$h = \frac{u^2 \sin^2\theta}{2a} = \frac{100 \times 100 \times \left(\frac{1}{2}\right)^2}{2 \times \frac{1 \times 10}{1}} = 125 \text{ m}$$

4. $$a = \frac{qE}{m} \Rightarrow m \uparrow \Rightarrow a \downarrow \Rightarrow h = \frac{u^2 \sin^2\theta}{2 \times \frac{qE}{m}}$$

$$\Rightarrow h \propto \frac{m}{q} \text{ [tan u, } \theta \text{ \& E is constant]}$$

Comprehention# 3

1.



For first particle

$$a = \frac{v^2}{r} = \frac{(10^3)^2}{4 \times 10^{-10}} = 2.5 \times 10^{15} \text{ m/s}^2$$

2. Acceleration of second particle

$$= \frac{v^2}{r} = \frac{(2 \times 10^3)^2}{8 \times 10^{-10}} = \frac{4}{8} \times 10^{16} = 5 \times 10^{15} \text{ m/s}^2$$

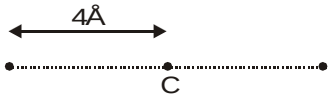
3. Velocity of centre of mass

$$= \frac{10^{-30} \times 2 \times 10^3}{2 \times 10^{-30} + 10^{-30}} = \frac{2}{3} \times 10^3 \text{ m / s}$$

4. Both particles move in a circular orbit about their centre of mass.

5. Angular velocity

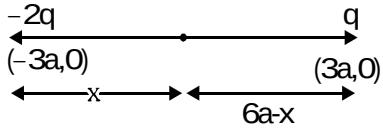
$$\omega = \frac{v}{r}$$



$$\omega = \frac{10^3}{4 \times 10^{-10}} = 2.5 \times 10^{12} \text{ rad/s}$$

Comprehention# 4

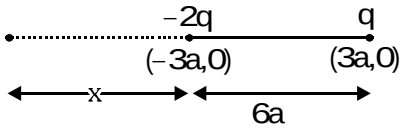
1. Let the potential will be zero at x distance from –2q.



$$\therefore \frac{Kx - 2q}{x} + \frac{Kq}{(6a - x)} = 0 \quad \therefore x = 4a$$

$$\frac{Kx - 2q}{x} + \frac{Kq}{6a + x} = 0 \Rightarrow x = -12a$$

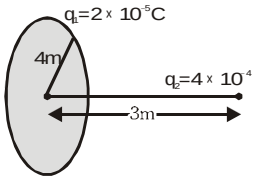
OR



2. At x = – 3a & x = 3a, the potential becomes – ∞ & + ∞ respectively and from the above question potential becomes zero at (a, 0) and (9a, 0)

Comprehention# 5

1.



From energy conservation

$$\frac{Kq_1q_2}{\sqrt{4^2 + 3^2}} + 0 + 0 = 2 \times \frac{1}{2} mv^2 + 0$$

$$v = \sqrt{\frac{9 \times 10^9 \times 2 \times 10^{-5} \times 4 \times 10^{-4}}{5}} = \frac{3 \times 2\sqrt{2}}{\sqrt{5}} = 3.1 \text{ m/s}$$

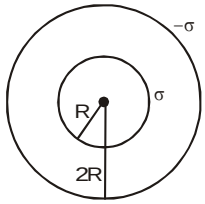
2. $$\frac{Kq_1q_2}{5} = \frac{1}{2} mv^2 \Rightarrow v = \frac{2}{\sqrt{5}}$$

Comprehention# 6

1. $Q_1 = \sigma \times 4\pi R^2$

$Q_2 = -\sigma \times 4\pi (2R)^2 = -4Q_1$

Potential at outer shell



$$= \frac{K[Q_1 - Q_2]}{2R} = \frac{K[Q_1 - 4Q_1]}{2R}$$

$$= \frac{-1 \times 3 \times \sigma \times 4\pi R}{4\pi \epsilon_0 \times 2R} = -\frac{3}{2}\frac{\sigma}{\epsilon_0} \times R$$

2. Electric field (r > 2R) before connecting the shells

$$E_1 = \frac{1 \times \sigma \times 4\pi R^2}{4\pi \epsilon_0 \times r^2} + \frac{1(-\sigma) \times 4\pi R^2 \times 4}{4\pi \epsilon_0 \times r^2} = \frac{\sigma}{\epsilon_0}\left[\frac{R^2}{r^2} - \frac{4R^2}{r^2}\right]$$

After connecting

$$E_2 = \frac{1}{4\pi \epsilon_0}\frac{[Q_1 - Q_2]}{r^2} = \frac{\sigma}{\epsilon_0}\left[\frac{R^2}{r^2} - \frac{4R^2}{r^2}\right]$$

$\therefore E_1 / E_2 = 1$

3. Electric field at $r = \frac{3R}{2}$ [Before connecting]

$$E_1 = \frac{1}{4\pi \epsilon_0} \times \frac{\sigma 4\pi R^2}{\left(\frac{3R}{2}\right)^2} + 0$$

After connecting $E_2 = 0$.

Hence $\left|\frac{E_2}{E_1}\right| = 0$

Comprehention# 7

1. $E = \frac{KQ}{r^2} \Rightarrow Q = \frac{Er^2}{K} = 13\mu C$

Comprehention# 8

1. P.E. = $\frac{1}{4\pi \epsilon_0}\frac{Q_e}{r}$ and KE = $\frac{1}{2}mv^2$

Hence total energy is greater than $\frac{eQ}{4\pi \epsilon_0 r}$

2. After long time sphere gets positive charge so the trajectory of the proton is (4) due to repulsion.

3. From the angular momentum conservation

$mv_0 \times \frac{R}{2} = mv \times R \Rightarrow v = \frac{v_0}{2}$

4. Limiting electric potential = change in ΔKE

$(KE)_i = \frac{1}{2}mv_0^2$; $(KE)_f = \frac{1}{2}m\left(\frac{v_0}{2}\right)^2$

$\therefore$ Electric potential = $\frac{3mv_0^2}{8e}$

5. From previous questions we can see that the final potential energy of the sphere is equal to the 3/4 of initial kinetic energy

$\Rightarrow \frac{3}{4} \times 2.56$ = 1.92 kV

Comprehention# 9

1. $E = \frac{-dV}{dr}$; $\frac{\sigma}{\epsilon_0} = \frac{-30}{0.3}$

$\sigma = 8.85 \times 10^{-12} \times 10^2$ = 8.85×10^{-10} C/m^2

2. Positive charge has a tendency to move from higher potential to lower potential hence it will move from B (–20V) to A (–30V).

3. Here $V_{DE} = V_D - V_E = 20 - (-20) = 40V$

Work done = qV_{DE}

$= -1 \times 10^{-6} \times 40$

$= -4 \times 10^{-5}$ J

Comprehention# 10

1. As given in paragraph, it is treated as +q and –q point charge at a distance 2a

$$F = \frac{1}{4\pi \epsilon_0}\frac{(-q)(q)}{(2d)^2}$$

2. Potential energy= $-\int \vec{F} \cdot d\vec{r}$

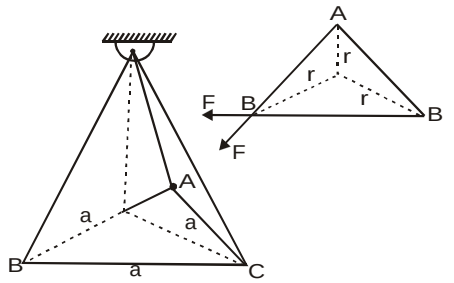
$$= +\int_{\infty}^{d} \frac{q^2}{4\pi \epsilon_0 (2r)^2} dr = -\frac{1}{4\pi \epsilon_0}\frac{q^2}{4d}$$

Note : Here potential energy $\neq -\frac{1}{4\pi \epsilon_0}\frac{q^2}{2d}$ because it is not an electrostatic field.

3. Work done by external force is change in potential energy .

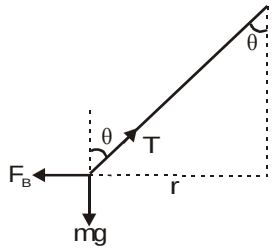
EXERCISE –IV(A)

1.



$F_B = 2F\cos 30° = \frac{2Kq^2}{a^2} \times \frac{\sqrt{3}}{2} = \frac{\sqrt{3}Kq^2}{a^2}$

For Charge at B : $2r\cos 30° = a$



$r = \frac{a}{\sqrt{3}}$

$T\sin\theta = F_B$

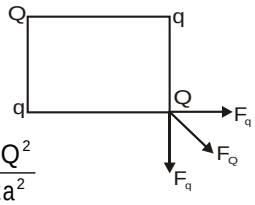
$T\cos\theta = mg$

$\tan\theta = \frac{F_B}{mg}$

$\Rightarrow \frac{\sin\theta}{\cos\theta} = \frac{99\times10^{-2}}{\sqrt{3}\times10^{-2}} = \frac{\sqrt{3}\times9\times10^{9}\times q^2}{(3\times10^{-2})2} \times \frac{1}{10^{-3}\times10}$

$\Rightarrow q = 3.17 \times 10^{-9}C$

2. $(F_Q)_{total} = \left|\vec{F}_{Qq} + \vec{F}_{Qq} + \vec{F}_{qQ}\right|$

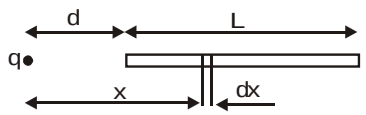


$O = 2F_q\cos 45° + F_Q$

$O = 2 \times \frac{1}{\sqrt{2}} \times \frac{KQq}{a^2} + \frac{KQ^2}{2a^2}$

$q = \frac{-Q}{2\sqrt{2}} = \frac{\sqrt{2}}{2\sqrt{2}} = -\frac{1}{2}\mu C$

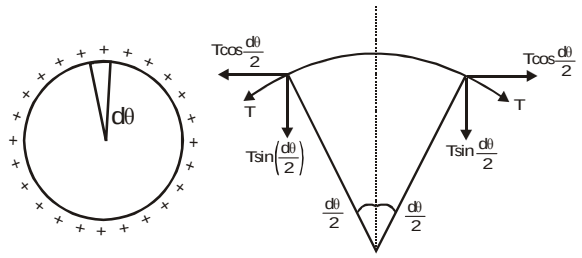
3. We consider an element of thickness dx at a distance x from q



Force on q = $\int \frac{1}{4\pi\epsilon_0} \times \frac{dQ.q}{x^2}$

Where $dQ = \frac{Q}{L}dx$ then ,$F = \frac{KQq}{L}\int_d^{d+L}\frac{dx}{x^2} = \frac{KQq}{d(d+L)}$

4. We consider an element subtending angle $d\theta$ at the centre

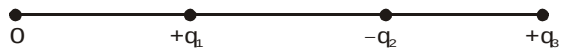


F = electrostatic force on element

$= \frac{KdQ.q}{R^2} = \frac{\left(\frac{KQ}{2\pi R}\times Rd\theta\right)}{R^2}q = \frac{KQq}{2\pi R^2}d\theta$

$2T\sin\frac{d\theta}{2} \simeq 2T\times\frac{\theta}{2} = \frac{KQqd\theta}{2\pi R^2} \therefore T = \frac{Qq}{8\pi^2\epsilon_0 r^2}$

5.



$q_1 = +10\ \mu C = q_3 = q_5$

$q_2 = q_4 = q_6 = -10\ \mu C$

$r_1 = 1m$

$r_2 = 3m$

$r_3 = 9\ m = 3^2 m$

$E_{at\ 0} = \frac{Kq_1}{r_1^2} - \frac{Kq_2}{r_2^2} + \frac{Kq_3}{r_3^2} - \frac{Kq_4}{r_4^2} + ...$

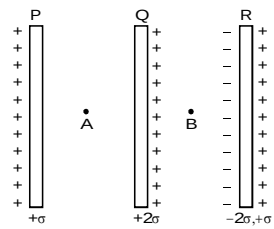
$= Kq\left[\frac{1}{1^2} - \frac{1}{3^2} + \frac{1}{3^4} - \frac{1}{3^6} + ...\right]$

$= \frac{Kq}{1-\frac{1}{3^2}} = \frac{9}{8}Kq = \frac{9}{8} \times 9 \times 10^9 \times 1 \times 10^{-6}$

$= \frac{8.1}{8}\times10^4 N/C$

6. Charge on outer plates = $\frac{\sigma A + 2\sigma A - \sigma A}{2} = \sigma A$

Charge distribution

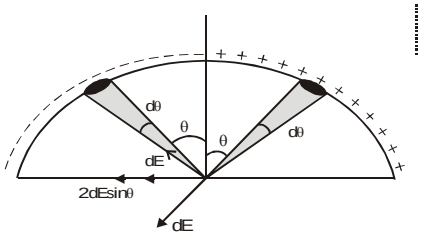


Field at A : $E_A = 0$

Field at B :

$E_B = \frac{2\sigma}{\varepsilon_0} \therefore \frac{E_A}{E_B} = 0$

7.



$$E_{net} = 2\int_0^{\pi/2} dE \sin\theta$$

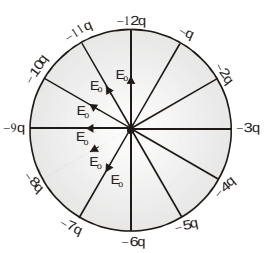
$$\Rightarrow 2\int_0^{\pi/2} \frac{\frac{Kq}{(\pi R/2)} \times R d\theta}{R^2} \times \sin\theta = \frac{4Kq}{\pi R^2}$$

$$\Rightarrow \vec{E} = \left(\frac{4Kq}{\pi R^2}\right)(-\hat{i})\, N/C$$

8. $E_x = E_0 + 2E_0 \cos 30° + 2E_0 \cos 60° = \left(2+\sqrt{3}\right)E_0$

$E_y = E_0$; $\tan\theta = \frac{E_y}{E_x} = \frac{1}{2+\sqrt{3}} \times 2-\sqrt{3}$; $\theta = 15°$

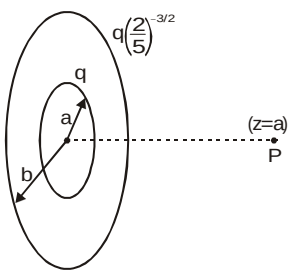
The hour hand will point towards 9 : 30



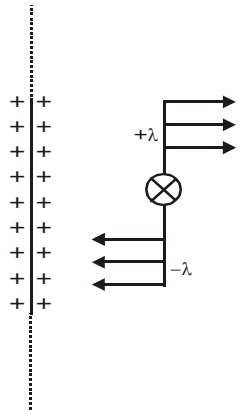
9. The charge at 'P' is in equilibrium.

Hence $E_P = 0$; $\vec{E}_P = \vec{E}_{Pa} + \vec{E}_{Pb} = \vec{0}$

$$\frac{Kqa}{(a^2+a^2)^{3/2}} - \frac{Kq\left(\frac{2}{5}\right)^{-3/2} a}{(a^2+b^2)^{3/2}} = 0 \Rightarrow \frac{b}{a} = 2$$



10.



Net force on upper half $= \left(\frac{\lambda\ell}{2}\right)\left(\frac{\sigma}{2\epsilon_0}\right)$

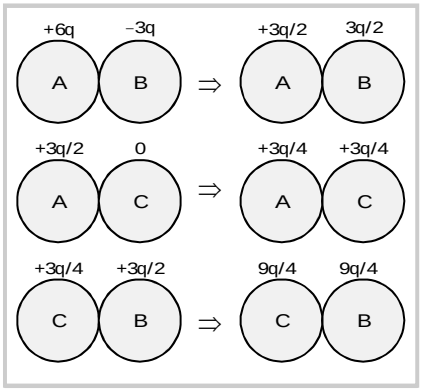
Torque on upper half $= \left(\frac{\lambda\ell}{2}\right) \times \frac{\sigma}{2\epsilon_0} \times \frac{\ell}{4}$

[$\frac{\ell}{4}$ = distance from hinge point for centre of mass]

Total torque on rod

$$= \frac{\lambda\ell}{2} \times \frac{\sigma}{2\epsilon_0} \times \frac{\ell}{4} \times 2 = I\alpha = \frac{m\ell^2}{12}\alpha \quad \therefore \alpha = \frac{3\lambda\sigma}{2m\epsilon_0}$$

11.



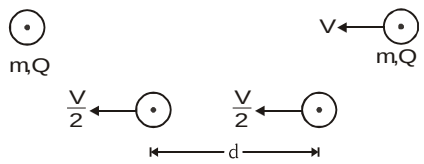
12. $U_{total} = U_{12} + U_{13} + U_{14} + U_{23} + U_{24} + U_{34}$

$$U_{initial} = \frac{Kq^2}{a} + \frac{Kq^2}{a} + \frac{Kq^2}{\sqrt{2}a} + \frac{Kq^2}{a} + \frac{Kq^2}{\sqrt{2}a} + \frac{Kq^2}{a}$$

$$U_{final} = \frac{Kq^2}{\sqrt{2}a} + \frac{Kq^2}{\sqrt{2}a} + \frac{Kq^2}{2a} + \frac{Kq^2}{\sqrt{2}a} + \frac{Kq^2}{2a} + \frac{Kq^2}{\sqrt{2}a}$$

$$\Delta U = U_f - U_i = -\left(3-\sqrt{2}\right)\frac{Kq^2}{a}$$

13.



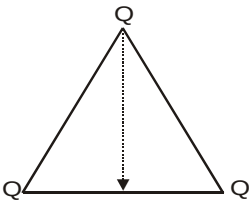
Conservation of momentum (COM)

$0 + mV = (m+m)\,V_f,\ V_f = \frac{V}{2}$

COME : $\frac{1}{2}mV^2 = \frac{1}{2}m\left(\frac{V}{2}\right)^2 + \frac{1}{2}m\left(\frac{V}{2}\right)^2 + \frac{KQ^2}{d}$

$d = \frac{Q^2}{\pi m \epsilon_0 v^2}$

14. $U_{initial} = \frac{KQ^2}{a} \times 2 = \frac{2KQ^2}{a}$

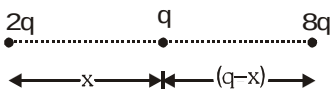


$U_{final} = \frac{KQ^2}{\left(\frac{a}{2}\right)} \times 2 = \frac{4KQ^2}{a}$

$\Delta U = \frac{2KQ^2}{a} = 1.8 \times 10^8$ J = $P \propto t$

$t = \frac{1.8 \times 10^8}{10^3}$ sec = 1.8×10^5 sec

15. For minimum potential energy of the system, q should be placed in the middle



$U_{system} = \frac{K2q \times q}{x} \times 100 + \frac{Kq8q}{(9-x)} \times 100$

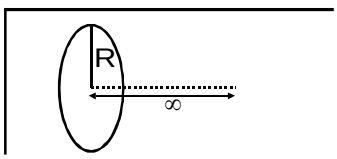
$+ \left(\frac{K \times 2q \times 8q}{9}\right) \times 100$

$\frac{dU}{dx} = Kq^2 \times 100 \left[\frac{-2}{x^2} + \frac{8}{(9-x)^2} + 0\right] \Rightarrow x = 3$ cm

Electric field at the position of

$q = \frac{K \times 2q}{(0.03)^2} - \frac{K \times 8q}{(0.06)^2} = 0$

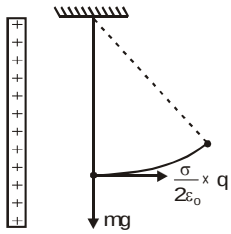
16. COME : $K_i + U_i = K_f + U_f$



$0 + 0 = \frac{1}{2}mV^2 - \frac{KQ^2}{R}$

$V = \sqrt{\frac{2KQ^2}{mR}}$

17. From work energy theorem (WET)



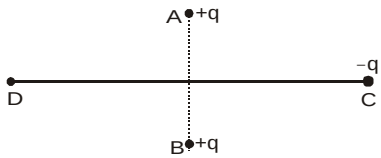
$W_{EF} + W_{mg} = \Delta KE$

$q_0 \times \frac{\sigma}{2\epsilon_0} \times L\sin\theta - mgL(1-\cos\theta) = 0$

$= q_0 \times \frac{\sigma}{2\epsilon_0} \times L \times 2\sin\frac{\theta}{2}\cos\frac{\theta}{2}$

$= mgL \times 2\sin^2\frac{2\theta}{2}$; $\tan\frac{\theta}{2} = \frac{\sigma}{2\epsilon_0} \times \frac{q_0}{mg}$

18.



COME : $K_C + U_C = K_D + U_D$

$4 + 2\left[\frac{Kq \times (-q)}{5}\right] = 0 + \left(\frac{Kq \times -2}{r}\right)$

$4 - \frac{2 \times 9 \times 10^9 \times 25 \times 10^{-10}}{5}$

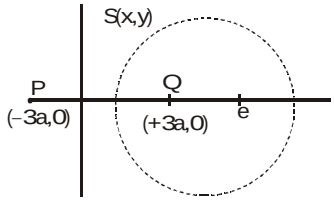
$= \frac{-2 \times 9 \times 10^9 \times 25 \times 10^{-10}}{r}$; r = 9 m

OD = $\sqrt{AD^2 - OA^2} = \sqrt{9^2 - 3^2} = 8.48$m

19. $V_A = \frac{KQ}{2} - \frac{KQ}{3}$, $V_B = \frac{KQ}{3} - \frac{KQ}{2}$ $\Big|$ $V_A - V_B = 2\left(\frac{KQ}{2} - \frac{KQ}{3}\right)$

$= 9 \times 10^9 \times 10^{-6}\left(1 - \frac{2}{3}\right) = 3000$ volt

20.



[A] $V_S = 0 = \frac{KQ}{\sqrt{(x-3a)^2 + y^2}} - \frac{2KQ}{\sqrt{(x+3a)^2 + y^2}}$

$\Rightarrow (x-5a)^2 + y^2 = (4a)^2 \therefore C = (5a, 0)$ and radius = 4a

[B] $V_{(x)} = \frac{KQ}{x-3a} - \frac{2KQ}{x+3a}$ for x > 3a

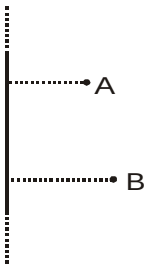
$$= \frac{KQ}{(3a-x)} - \frac{2KQ}{3a+x} \text{ for } x < 3a$$

[C] A positive charge when released will move from high potential to low potential

COME : $K_1 + U_1 = K_2 + U_2$

$$0 + q\left[\frac{KQ}{2a} - \frac{2KQ}{8a}\right] = \frac{1}{2}mv^2 + 0 \Rightarrow V = \sqrt{\frac{Qq}{8\pi \epsilon_0 ma}}$$

21. Force at A $\frac{\lambda q}{2\pi \epsilon_0 r} = 100$



$$\frac{\lambda q}{2\pi \epsilon_0 \times 0.2} = 100 \quad \int_A^B dv = -\int_A^B \vec{E}.dr$$

$$V_B - V_A = \frac{-\lambda}{2\pi \epsilon_0}\int_{r_1}^{r_2}\frac{dr}{r} = \frac{-\lambda}{2\pi \epsilon_0}\ell n\frac{r_2}{r_1}$$

$$V_A - V_B = \frac{\lambda}{2\pi \epsilon_0}\ell n\frac{r_2}{r_1} = \frac{\lambda}{2\pi \epsilon_0}\ell n2$$

COME : $K_A + U_A = K_B + U_B$

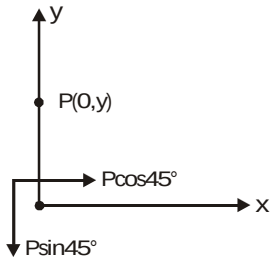
$$0 + \frac{\lambda Q}{2\pi \epsilon_0}\ell n2 = \frac{1}{2}mv^2 ; 20\ \ell n2 = \frac{1}{2} \times 0.1 \times V^2$$

$$V = 20\sqrt{\ell n2}$$

22. $\vec{E}_x = \frac{-K(P\cos 45°)}{y^3}\hat{i}$

$\vec{E}_y = \frac{-2x(P\sin 45°)}{y^3}\hat{j}$

$\vec{E}_P = \frac{KP}{\sqrt{2}y^3}\left[-\hat{i} - 2\hat{j}\right]$



23. Electric field due to dipole $\vec{P} = -\frac{KP}{1^3} \times \hat{k}$

Electric field due to dipole $\frac{\vec{P}}{2} = +\frac{2K \times \frac{P}{2}}{2^3}\hat{k}$

Resultant electric field = $-\frac{7}{8}KP\hat{k}$

24. $|V_P| = |E_P|$

$$\frac{KP\cos\theta}{r^2} = \frac{KP}{r^3}\sqrt{1+3\cos^2\theta} ; \cos\theta = \frac{1}{\sqrt{r^2-3}}$$

$r^2 - 3 \geq 1$; $r \geq 2$

Now $\cos\theta = \frac{1}{\sqrt{r^2-3}}\left(r = \sqrt{5}\right) = \frac{1}{\sqrt{2}}$

then θ = 45°, 135°, 225°, 315°

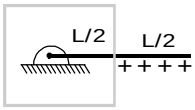
25. Eight cubes encloses a charge. For a cube, the three surfaces meeting the charge contribute to zero flux. Hence the rest three surfaces including the shaded surface have equal flux passing through them and are equal to $\frac{q}{24 \epsilon_0}$.

26. Flux = $\frac{q}{4 \epsilon_0} = \frac{q}{\epsilon_0}\frac{(1-\cos\theta)}{2}$

where θ is the semi–vertex angle

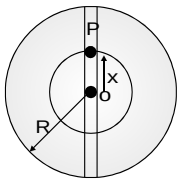
$$\Rightarrow \cos\theta = \frac{1}{2} = \frac{a}{\sqrt{a^2+R^2}} ; a = \frac{R}{\sqrt{3}}$$

27.



Minimum flux through cube = $\frac{Q}{2 \epsilon_0}$

28.

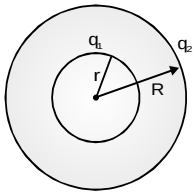


Force on charge at position P

$$= -\left(\frac{1}{4\pi \epsilon_0} \times \frac{Q}{R^3}x\right)q = -m\omega^2 x$$

$$T = 2\pi\sqrt{\frac{qQ}{4\pi \epsilon_0 \times mR^3}}$$

29. $q_1 + q_2 = Q$

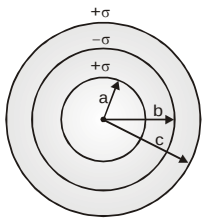


$$\sigma 4\pi r^2 + \sigma 4\pi R^2 = Q$$

$$\sigma = \frac{Q}{4\pi(r^2+R^2)}$$

$$V_{centre} = \frac{1}{4\pi \epsilon_0}\left[\frac{\sigma 4\pi r^2}{r} + \frac{\sigma 4\pi R^2}{R}\right] = \frac{Q(r+R)}{4\pi \epsilon_0 (r^2+R^2)}$$

30.



(i) $V_A = \frac{1}{4\pi \epsilon_0}\left[\frac{4\pi a^2\sigma}{a} - \frac{4\pi b^2\sigma}{b} + \frac{4\pi c^2\sigma}{c}\right]$

$$= \frac{\sigma}{\epsilon_0}(a-b+c)$$

$$V_B = \frac{1}{4\pi\epsilon_0}\left[\frac{4\pi a^2\sigma}{b} - \frac{4\pi b^2\sigma}{b} + \frac{4\pi c^2\sigma}{c}\right]$$

$$V_C = \frac{1}{4\pi\epsilon_0}\left[\frac{\sigma 4\pi a^2}{c} - \frac{\sigma 4\pi b^2}{c} + \frac{\sigma 4\pi c^2}{c}\right]$$

$$= \frac{\sigma}{\epsilon_0}\left(\frac{a^2}{c} - \frac{b^2}{c} + c\right)$$

(ii) $V_A = V_C$; $\frac{\sigma}{\epsilon_0}(a-b+c) - \frac{\sigma}{\epsilon_0}\left[\frac{a^2}{c} - \frac{b^2}{c} + c\right]$; $c = (a+b)$

31. From mass conservation

$$27 \times \left(\frac{4}{3}\pi r^3\right) = \frac{4}{3}\pi R^3 ;\ R = 3r$$

Given that $V_0 = \frac{Kq}{r}$ then $V_{bigdrop}$

$$= \frac{K \times (27q)}{R} = \frac{K(27q)}{3r} = \frac{9Kq}{r} = 9V_0$$

32. After earthing $V_{inner\ shell} = 0$; $\frac{Kq}{R} + \frac{KQ}{3R} = 0$

$q = \frac{-Q}{3}$ (charge on inner shell)

33. $(U_{self})_{initial} = \frac{1}{4\pi\epsilon_0}\left(\frac{Q^2}{2R}\right)$

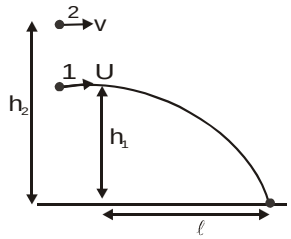
$(U_{self})_{final} = \frac{1}{4\pi\epsilon_0}\left(\frac{Q^2}{4R}\right)$

Workdone against electric forces = ΔU

$$= -\frac{1}{4\pi\epsilon_0}\frac{Q^2}{4R} = -\frac{\pi\sigma^2 R^3}{\epsilon_0}$$

EXERCISE –IV(B)

1.



Time to fall first ball

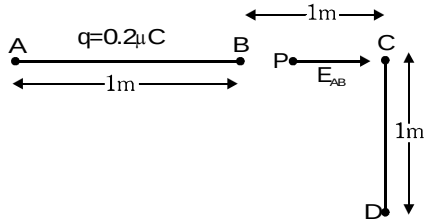
$t = \frac{\ell}{v}$ $\therefore h_1 = \frac{1}{2} \times g \times \frac{\ell^2}{v^2}$

$\therefore$ Height of 2nd ball

$$= h_2 - \frac{1}{2}g\frac{\ell^2}{v^2}$$

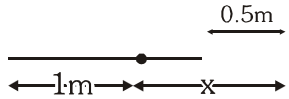
$$pr = h_2 + \frac{1}{2}g\frac{\ell^2}{v^2} - g\frac{\ell^2}{v^2};\ h_2 + h_1 - g\left(\frac{\ell^2}{v^2}\right)$$

2.



$$\vec{E}_{AB} = \int_{0.5}^{1.5}\frac{K\lambda dx}{x^2} = -K\lambda\left[\frac{2}{3} - 2\right] = \frac{\lambda}{3\pi\epsilon_0}$$

From the wire CD



$E_{\parallel} = \frac{K\lambda}{r}(\cos\theta_2 - \cos\theta_1)$ and

$\{\theta_2 = 0, \theta_1 = \tan^{-12}$ & $E_{\perp} = \frac{K\lambda}{r}[\sin\theta_2 + \sin\theta_1]$

3. Momentum of 1st ball

$P_i = mvi$; $P_f = m\frac{v}{2}(\cos 60°\hat{i} + \sin 60\hat{j})$

change in momentum

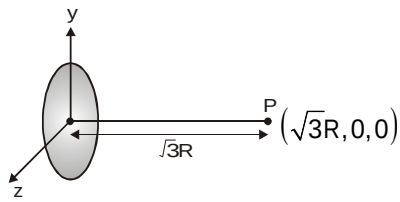
$$\Delta P = P_f - P_i = -\frac{3v}{4}\hat{i} + \frac{\sqrt{3}v}{4}\hat{j}$$

Momentum of IInd ball :

$P_i = mv\hat{i}$ $\qquad$ $P_f = mv\hat{j}$

Change in momentum = $mv[\hat{j} - \hat{i}]$

4.



K.E. at 'P' must be sufficient to reach the charge particle

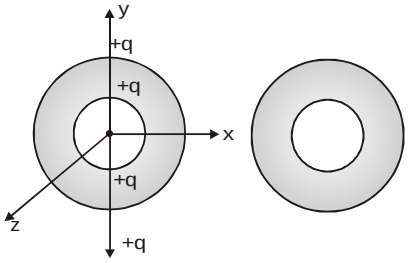
at the centre of the ring.

$(ME)_P = (ME)_{\text{centre of ring}}$

$$\frac{1}{2}mv^2 + \frac{K\pi\lambda \times 2\pi Rq}{\sqrt{\left(R^2+\sqrt{3}R\right)^2}} = O + \frac{K \times \lambda 2\pi Rq}{R}$$

$$v = \sqrt{\frac{\lambda q}{2\,\epsilon_0\, m}} \quad \left(\because K = \frac{1}{4\pi\,\epsilon_0}\right)$$

5.



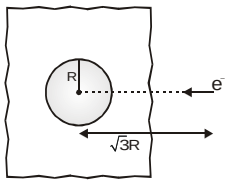
P.E. of outer ring charges does not change

$$= \frac{Kq^2}{R} \times 2 + \frac{Kq^2}{3R} \times 2 = \frac{8Kq^2}{3R}; \quad U_f = \frac{Kq^2}{\sqrt{5}R} \times 4 .$$

Hence $W_{electron} = -\Delta U$

$$W_{electron} = -(V_f - V_i) = -\frac{Kq^2}{R}\left[\frac{4}{\sqrt{5}} - \frac{8}{3}\right] = \frac{Kq^2}{R}\left[\frac{8}{3} - \frac{4}{\sqrt{5}}\right]$$

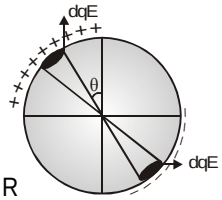
6. From the energy conservation



$$\left[\frac{\sigma R}{2\,\epsilon_0} - \frac{\sigma}{2\,\epsilon_0} \times 2R\right]e = \frac{1}{2}mv^2 -$$

$$-\frac{\sigma Re}{2\,\epsilon_0} + \frac{\sigma Re}{\epsilon_0} = \frac{1}{2}mv^2 \quad \sqrt{\frac{\sigma eR}{m\,\epsilon_0}}$$

8. $\int_0^{\frac{\pi}{2}} 2(\lambda R d\theta) \times E_0 R\cos\theta = F \times R$



$$2\lambda R^2 E_0 \int_0^{\frac{\pi}{2}} \cos\theta d\theta = 2\lambda R^2 E_0 = FR$$

$\Rightarrow F = 2\lambda RE_0$

9. $q = \epsilon_0 \int \vec{E}\cdot d\vec{S} = \epsilon_0 \int \frac{E_0 x}{\ell}\hat{i}$

$$\left(dx\hat{i} + dy\hat{j} + dz\hat{k}\right) = \epsilon_0 \times \left[\frac{E_0 x^2}{2\ell}\right]_0^a$$

Put given values $\Rightarrow 2.2 \times 10^{-12}$ C

10. [i] $r < a \Rightarrow$ Point inside $\Rightarrow$ E=0

[ii] $a < r < b \Rightarrow$ Point outside the inner and inside the outer so field due to only inner cylinder

$a < r < b \Rightarrow$

hence $E = \frac{2 \times \lambda}{r}$

[iii] $r > b \Rightarrow$ point outside from both cylinders so the charge per unit length (λ) is zero for point ($r > b$) $\therefore E = 0$

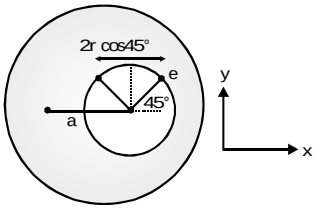
$r > b \Rightarrow$

11. (i) $q = \int dq = \int_0^R \rho_0 \frac{r}{R} \times 4\pi r^2 dr = \frac{\rho_0 \times 4\pi R^4}{R} = \pi\rho_0 R^3$

(ii) $E = \frac{KQ'}{r^2} = \frac{K}{r^2} \times \frac{\rho_0 \pi r^4}{R} = \frac{KQr^2}{R^4}$ where

$$Q' = \int_0^r \rho_0 \frac{x}{R} \times 4\pi x^2 dx = \frac{\rho_0}{4R} \times 4\pi \times r^4 = \frac{\rho_0 \pi r^4}{R}$$

12. The direction of electric field inside the cavity in –ve x direction and of constant magnitude $\frac{\rho a}{3\,\epsilon_0}$.

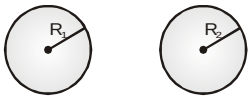


For touch the sphere again, electron must move 2r cos45° (as shown) distance

$$\therefore \frac{1}{2}\left[\frac{\rho a}{3\,\epsilon_0} \times \frac{e}{m}\right] \times t^2 = \sqrt{2}r$$

$$\therefore t = \sqrt{\frac{6\sqrt{2}mr\,\epsilon_0}{e\rho a}}$$

13.



Electrostatic energy of system = Interaction Energy + Self Energy

(Let the total charge of balls be Q)

$$U = 0 + \frac{KQ_1^2}{2R_1} + \frac{K(Q-Q_1)^2}{2R_2}$$

Here for it's minimum value $\frac{dU}{dQ_1} = 0$

$$= K\left[\frac{2Q_1}{2R_1} + \frac{2(Q-Q_1)}{2R_2}(0-1)\right] = 0$$

$$\Rightarrow \frac{Q_1}{R_1} - \frac{Q_2}{R_2} = 0 \quad [\because Q - Q_1 = Q_2] \Rightarrow \frac{Q_1}{Q_2} = \frac{R_1}{R_2}$$

14. From the energy conservation

$$\frac{KQq}{r}+\frac{1}{2}mv^2=\frac{11}{8}\frac{KQq}{R}+0$$

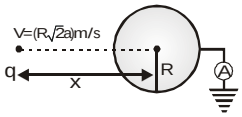
$$v=\sqrt{\frac{2KQq}{mR}\left[\frac{11}{8}-\frac{R}{r}\right]}$$

15. Let n^{th} number last drop that can entre

$$\frac{KQ^2}{R}\times n = mg\ (h-R) \quad \therefore n=\frac{4\pi\epsilon_0\, mg(h-R)R}{Q^2}$$

16. $\frac{Kq}{x}+\frac{KQ}{R}=0 \quad -\frac{Kq}{x^2}\frac{dx}{dt}+\frac{K}{R}\frac{dQ}{dt}=0$

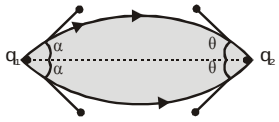
$$i=\frac{dQ}{dt}=\frac{qv}{x^2}\times R=\frac{q}{\left(\sqrt{2a}R\right)^2}\times R\sqrt{2a}\times R=\frac{q}{2a}$$



17. Solid angle = 2π $(1-\cos\alpha)$

$$\frac{q_1}{2\epsilon_0}(1-\cos\alpha)=\frac{q_2}{2\epsilon_0}(1-\cos\theta)$$

$$q_1\frac{\sin^2\alpha}{2}=q_2\frac{\sin^2\theta}{2}$$



$$\sin\frac{\theta}{2}=\sqrt{\frac{q_1}{q_2}}\sin\frac{\alpha}{2}$$

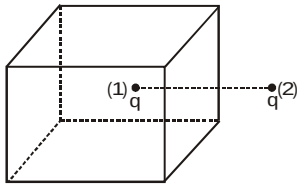
$$\theta=2\sin^{-1}\left[\sqrt{\frac{q_1}{q_2}}\sin\frac{\alpha}{2}\right]$$

EXERCISE-V-A

1. Work done = (charge) (P.D.)

$$\text{P.D.}=\frac{\text{Work done}}{\text{Charge}}=\frac{2J}{20C}=0.1V$$

2.



$$\phi=\frac{q}{6\epsilon_0}-\frac{q}{6\epsilon_0}=0$$

3. $V_P=\frac{Kq}{R}+\frac{KQ}{R/2}=\frac{2Q}{4\pi\epsilon_0 R}+\frac{q}{4\pi\epsilon_0 R}$

4. Let charges on B & C be Q then final charges on B and C will be $\frac{Q}{2}$ and $\frac{3Q}{4}$ respectively.

Therefore $F_{BC}=\frac{K(Q/2)(3Q/4)}{r^2}=\frac{3}{8}F$

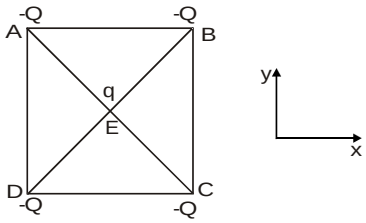
5. At the distance of closest approach

$K_{initial}$ = PE_{final}

$$\frac{1}{2}mv^2=\frac{Kq_1q_2}{r_0}\Rightarrow r_0\propto\frac{1}{v^2}$$

Hence, $\frac{r_{20}}{r_{10}}=\left(\frac{v_1}{v_2}\right)^2=\left(\frac{v}{2v}\right)^2=\frac{1}{4}\Rightarrow r_{20}=\frac{r_{10}}{4}$

6.



On establishing equilibrium on charge at A

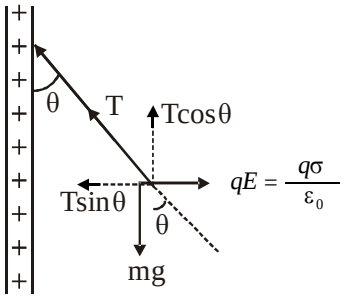
$$\vec{F}_{AC}=\frac{kQ^2}{\left(\sqrt{2}r\right)^2}\left[\cos 45°\left(-\hat{i}\right)+\sin 45°\hat{j}\right]$$

$$\vec{F}_{AB}=\frac{kQ^2}{r^2}\left(-\hat{i}\right);\ \vec{F}_{AD}=\frac{kQ^2}{r^2}\hat{j}$$

$$\vec{F}_{AE}=\frac{kQq}{\left(r/\sqrt{2}\right)^2}\left[\cos 45°\hat{i}+\sin 45°\left(-\hat{j}\right)\right]$$

On solving we get $q = +\frac{Q}{4}\left[1+2\sqrt{2}\right]$

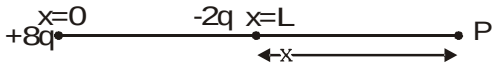
7.



$T\sin\theta = \frac{q\sigma}{\varepsilon_0}$ and $T\cos\theta = mg$

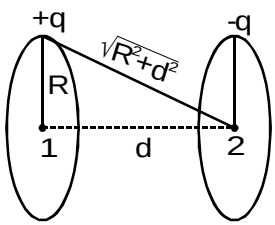
At equilibrium $\tan\theta = \frac{F_E}{mg} = \frac{q\sigma}{mg\varepsilon_0} \Rightarrow \sigma \propto \tan\theta$

8. Due to two charges of opposite nature, E will be zero on the line joining the two charges and nearer to the charge of smaller magnitude.



$$\frac{K8Q}{x^2} = \frac{K2q}{(x-L)^2} \Rightarrow x = 2L$$

9.

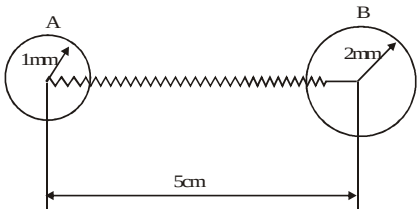


$$V_1 - V_2 = \frac{1}{4\pi\epsilon_0}\left[\left\{\frac{q}{R} + \frac{(-q)}{\sqrt{R^2+d^2}}\right\}\right] - \left\{\frac{(-q)}{R} + \frac{q}{\sqrt{R^2+d^2}}\right\}$$

$$= \frac{q}{2\pi\epsilon_0}\left[\frac{1}{R} - \frac{1}{\sqrt{R^2+d^2}}\right]$$

10. An electric dipole when made to interact with a non-uniform electric field in a direction other than the direction of the electric field, experiences force as well as torque.

11.



At equilibrium, the potential on the surface of both spheres will be same. Also, for equal potential

$$E \propto \frac{1}{\text{radius}} \Rightarrow \frac{E_A}{E_B} = \frac{r_B}{r_A} = \frac{2}{1}$$

12. Distance of point A from origin

$= \sqrt{2^2+0^2} = 2$ units

Distance of point B from origin

$= \sqrt{2^2+0^2} = 2$ units

As point A & B are situated at equal distance, so $v_A = v_B$

13. Potential remains same as it depends on algebraic sum of charges but electric field $\vec{E}$ changes.

14. $V(x) = \frac{20}{x^2-4}$

The electric field $\vec{E}$ along all such lines where potential is a function of position

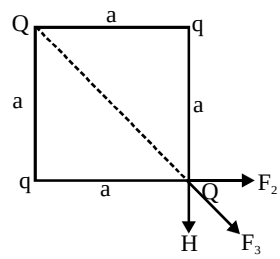
So $\frac{\partial V}{\partial x} = \frac{(x^2-4)(0)-20(2x)}{(x^2-4)^2} = \frac{-40x}{(x^2-4)^2}$

$\vec{E} = \frac{40x}{(x^2-4)^2}(\hat{i})$ At x = 4 μm

$\vec{E} = \frac{40(4)}{(4^2-4)^2} = \frac{160}{144} = \left(\frac{10}{9}\hat{i}\right)$ V / m

15. E(r) = 0 for $0 \le r < R$ = $\frac{Q}{4\pi\epsilon_0 r^2}$ for r > R

16. Here $\vec{F}_1 + \vec{F}_2 + \vec{F}_3 = 0$ ..... (i)



$\left|\vec{F}_1 + \vec{F}_2\right| = \frac{\sqrt{2}KQ^2}{a^2}$

$\left|\vec{F}_3\right| = \frac{KQ^2}{(\sqrt{2}a)^2}$

from (i)

$$\frac{\sqrt{2}KQq}{a^2} + \frac{KQ^2}{(\sqrt{2}a)^2} = 0$$

$$\frac{\sqrt{2}KQq}{a^2} = -\frac{KQ^2}{2a^2} \Rightarrow Q = -2\sqrt{2}\,q$$

18. $q = -100 \times 1.6 \times 10^{-19}$ C

$\Delta V = -14$volt

$W = q\Delta V = 2.24 \times 10^{-16}$ J

19. $Q' = \int \rho dV = \int_0^{r_1} \frac{Q}{\pi R^4} r.(4\pi r^2)dr = \frac{4Q}{R^4}\left[\frac{r_1^4}{4}\right]$

$$E = \frac{KQ'}{r_1^2} = \frac{1}{4\pi \epsilon_0 r_1^2}\left[\frac{4Qr_1^4}{4R^4}\right] = \frac{Qr_1^2}{4\pi \epsilon_0 R^4}$$

20. $\vec{E} = \frac{2k\lambda}{r}\left(-\hat{j}\right) = \frac{2}{\left(4\pi \epsilon_0 r\right)}\frac{q}{(\pi r)}\left(-\hat{j}\right)$

21. Total charge

$$Q = \int_0^r \rho\, dV = \int_0^r \rho_0\left(\frac{5}{4} - \frac{r}{R}\right)4\pi r^2 dr$$

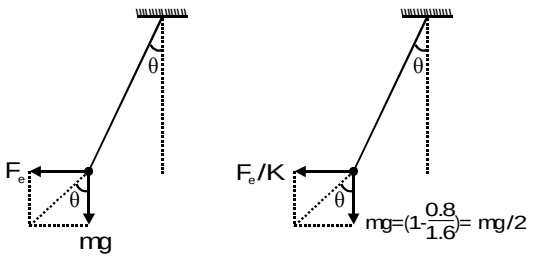
$$= 4\pi\rho_0\int_0^r\left(\frac{5r^2}{4} - \frac{r^3}{R}\right)dr$$

$$= 4\pi\rho_0\left[\frac{5r^3}{12} - \frac{r^4}{4R}\right]$$

$$E = \frac{KQ}{r^2} = \frac{1}{4\pi \epsilon_0 r^2}4\pi\rho_0\left[\frac{5}{12}r^3 - \frac{r^4}{4R}\right]$$

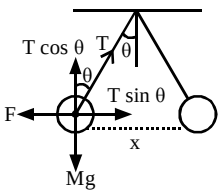
$$= \frac{\rho_0 r}{4\epsilon_0}\left[\frac{5}{3} - \frac{r}{R}\right]$$

22.



Therefore K =2

23. $\tan\theta = \frac{F}{Mg}$ (since θ small)



$\theta = \frac{F}{Mg} \Rightarrow F = Mg\theta$

$$\frac{KQ^2}{x^2} = Mg\frac{x}{\ell}$$

$$Q^2 = \frac{Mg}{K\ell}x^3 \Rightarrow Q = \sqrt{\frac{Mg}{K\ell}}\,x^{3/2}$$

$$\left(\frac{dQ}{dt}\right) = \sqrt{\frac{Mg}{K\ell}}\left(\frac{3}{2}x^{1/2}\right)\left(\frac{dx}{dt}\right) = \text{constant}$$

so $\frac{dx}{dt} \propto x^{-1/2}$ Or $V \propto x^{-1/2} \Rightarrow v \propto x^{-1/2}$

24. $E_r = -\frac{\partial\phi}{\partial r} = -2ar$

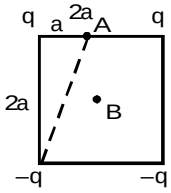
By gauss theorem $4\pi r^2 E_r = \frac{q}{\varepsilon_0}$

By differention $4\pi d\,(r^2E_r) = \frac{dq}{\varepsilon_0} = \frac{(4\pi r^2 dr)\rho}{\varepsilon_0}$

$\Rightarrow r^2dE_r + 2rE_r\,dr = \frac{1}{\varepsilon_0}\rho r^2 dr$

$\Rightarrow \frac{\partial E_r}{\partial r} + \frac{2}{r}E_r = \frac{\rho}{\varepsilon_0} \Rightarrow \rho = -6\varepsilon_0 a$

25. $\Delta K = W = Q(V_A - V_B)$



$$= Q\left[\left(\frac{2kq}{a} - \frac{2kq}{a\sqrt{5}}\right) - 0\right]$$

$$= \frac{2KqQ}{a}\left(1 - \frac{1}{\sqrt{5}}\right)$$

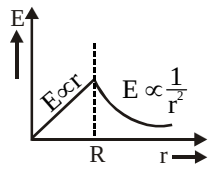
27. For uniformly charged sphere

$E = \frac{Kqr}{R^3}$ (r < R)

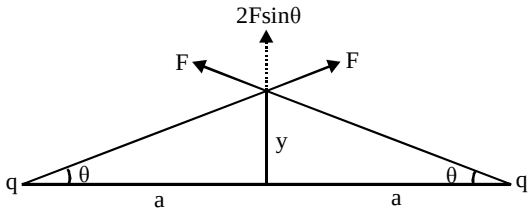
$E = \frac{Kq}{R^2}$ (r = R)

$E = \frac{Kq}{r^2}$ (r > R)

so

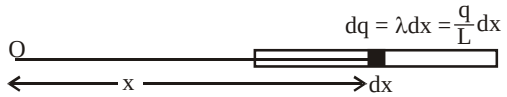


28. Net force $= 2F\sin\theta = 2\left[\frac{kqq_0}{\left(a^2+y^2\right)}\right].\frac{y}{\left(a^2+y^2\right)^{1/2}}$



$$= \frac{2kqq_0}{\left(a^2+y^2\right)^{3/2}} \approx \frac{2kqq_0 y}{a^3} \propto y$$

29. Potential at $O = \int_L^{2L}\frac{K\left(\frac{Q}{L}dx\right)}{x} = \frac{\theta}{4\pi\epsilon_0 L}\ln(2)$



EXERCISE –V(B)

1. Potential at origin will be given by

$$V = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{x_0} - \frac{1}{2x_0} + \frac{1}{3x_0} - \frac{1}{4}x_0 + \ldots\right]$$

$$= \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{x_0}\right)\left[1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \ldots\right]$$

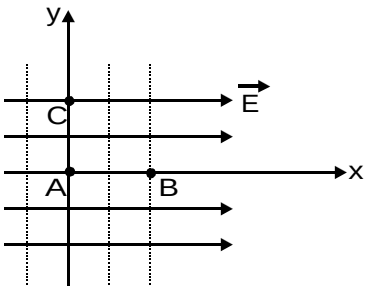
$$= \frac{q}{4\pi\varepsilon_0 x_0}\ell n(2)$$

2. Net electrostatic energy of the configuration will be

$$U = K\left[\frac{q.q}{a} + \frac{Q.q}{\sqrt{2}a} + \frac{Q.q}{a}\right] = O \Rightarrow Q = \frac{-2q}{2+\sqrt{2}}$$

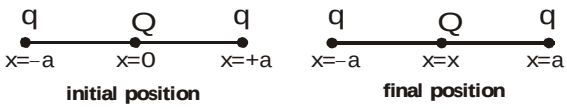
3. Electric lines of force never form a closed loop. Therefore, options (B) and (D) are wrong. Electric lines of force emanate from positive charge and terminate on negative charge, therefore, option(A) is also wrong.

4. Potential decreases in the direction of electric field. Dotted lines are equipotential lines.



$\therefore V_A = V_C$ and $V_A > V_B$

5.



$$U_i = \frac{2KQq}{a} \text{ and } U_f = KQq\left[\frac{1}{a+x} + \frac{1}{a-x}\right]$$

$$\Rightarrow |\Delta U| = \frac{2KQqx^2}{a^3} \text{ for } x<<a \quad \therefore \Delta U \propto x^2$$

6. Electric field is zero everywhere inside a metal (conductor) i.e., field lines do not enter a metal. Simultaneously these are perpendicular to a metal surface (equipotential surface).

7. According to option (d) the electric field due to P and S and due to Q and T add to zero. While due to U and R will be added up. Hence, the correct option is (D).

8. At any point over the spherical Gaussian surface, net electric field is the vector sum of electric fields due to $+q_1$, $-q_1$ and q_2.
Don't confuse with the electric flux which is zero (net) passing over the Gaussian surface as the net charge enclosing the surface is zero.

9. All the three plates will produce electric field at P along negative z–axis. Hence,

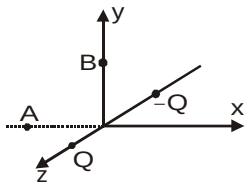
$$\vec{E}_p = \left[\frac{\sigma}{2\varepsilon_0} + \frac{2\sigma}{2\varepsilon_0} + \frac{\sigma}{2\varepsilon_0}\right](-\hat{k}) = -\frac{2\sigma}{\varepsilon_0}\hat{k}$$

10. There will be an electric field between two cylinders (using Gauss theorem). This electric field will produce a potential difference.

11. Charge will be induced in the conducting sphere, but net charge on it will be zero.

12. Inside the cavity, field at any point is uniform and non–zero.

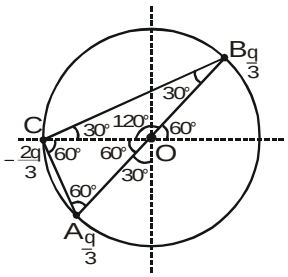
13. $A \equiv (-a,0,0), B \equiv (0,a,0)$



Point charge is moved from A to B

$V_A = V_B = 0 \therefore W=0$

14. $BC = 2R\sin\left(\frac{120}{2}\right) = \sqrt{3}\,R$



- Electric field at O

$$= \frac{1}{4\pi \in_0}\left(\frac{2q/3}{R^2}\right) = \frac{q}{6\pi \in_0 R^2}$$

along negative X-axis.

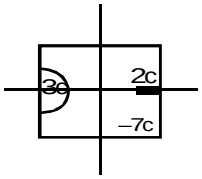
- The potential energy of the system is non zero
- Force between B & C B

$$= \frac{1}{4\pi \in_0}\frac{(q/3)(-2q/3)}{\left(\sqrt{3}R\right)^2} = -\frac{q^2}{54\pi \in_0 R^2}$$

- Potential at O

$$= \frac{1}{4\pi \in_0}\left(\frac{q}{3}+\frac{q}{3}-\frac{2q}{3}\right) = 0$$

15.



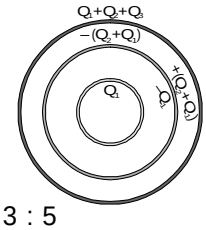
$$\phi = \frac{q_{inclosed}}{\in_0}$$

$$\phi = \frac{-2C}{\in_0}$$

16. $Q_1 = \sigma 4\pi R^2 = k$ (say)

$Q_1 + Q_2 = \sigma 4\pi\,(4R^2) = 4k$

$Q_1+Q_2+Q_3 = \sigma(4\pi)\,(9r^2) = 9k$



On solving, $Q_1 : Q_2 : Q_3 :: 1 : 3 : 5$

17. Since torque about central charge is zero angular momentum is conserved.

MCQ's

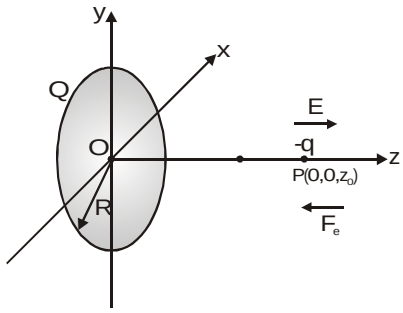
1. Let Q be the charge on the ring, the negative charge –q is released from point P (0,0, z_0). The electric field at P due to the charged ring will be along positive z–axis and its magnitude will be

$$E = \frac{1}{4\pi\varepsilon_0}\frac{Qz_0}{\left(R^2+z_0^2\right)^{3/2}}$$

E = 0 at centre of the ring because z_0 =0

Therefore, force on charge P will be towards centre as shown, and its magnitude is

$$F_e = qE = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Qq}{\left(R^2+z_0^2\right)^{3/2}}\cdot z_0 \quad ..(i)$$



Similarly, when it courses the origin, the force is again towards centre O.

Thus, the motion of the particle is periodic for all values of z_0 lying between 0 and ∞.

Secondly, if $z_0 << R$, $(R^2 + z_0^2)^{3/2} \approx R^3$

$$F_e \approx -\frac{1}{4\pi\varepsilon_0}\cdot\frac{Qq}{R^3}\cdot z_0 \text{ (From Eq. 1)}$$

i.e., the restoring force $F_e \propto -z_0$. Hence, the motion of the particle will be simple harmonic. (Here negative sign implies that the force is towards its mean position).

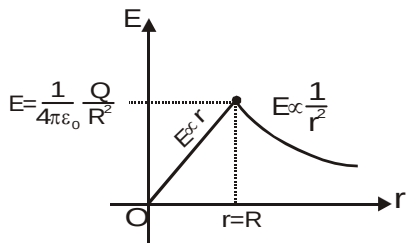
2. Inside the sphere $E = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q}{R^3}r \Rightarrow E \propto r$ for $r \le R$

i.e., E at centre = 0 (r=0)

and E at surface = $\frac{1}{4\pi\varepsilon_0}\cdot\frac{Q}{r^2}$ (r = R)

Outside the sphere $E = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q}{r^2}(r \ge R)$ or $E \propto \frac{1}{r^2}$

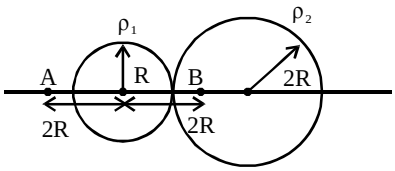
Thus, variation of electric field (E) with distance (r) from the centre will be as follows :



3. Under electrostatic condition, all points lying on the conductor are in same potential. Therefore, potential at A=potential at B. Hence, option (C) is correct. From Gauss theorem, total flux through the surface of the cavity will be $\frac{q}{\varepsilon_0}$.

4. The given graph is of charged conducting sphere of radius R_0. The whole charge q distributes on the surface of the sphere.

5. $q_1 = \rho_1 \frac{4}{3}\pi R^3$; $q_2 = \rho_2 \frac{4}{3}\pi (2R)^3$



if E_{net} = 0 at point A then

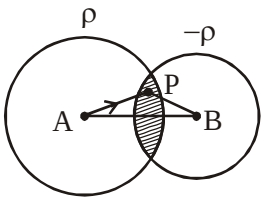
$$\frac{kq_1}{(2R)^2} + \frac{kq_2}{(5R)^2} = 0 \Rightarrow \frac{q_1}{q_2} = -\frac{4}{25} \Rightarrow$$

$$\frac{\rho_1}{\rho_2} = -\frac{32}{25}$$

if E_{net} = 0 at point B then

$$\frac{kq_1}{(2R)^2} = \frac{kq_2 R}{(2R)^3} \Rightarrow \frac{q_1}{q_2} = \frac{1}{2} \Rightarrow \frac{\rho_1}{\rho_2} = 4$$

6.



Electric field at point P

$$= \frac{\rho(\overrightarrow{AP})}{3\,\epsilon_0} + \frac{-\rho(\overrightarrow{BP})}{3\,\epsilon_0} = \frac{\rho(\overrightarrow{AP} - \overrightarrow{BP})}{3\,\epsilon_0} = \frac{\rho(\overrightarrow{AB})}{3\,\epsilon_0}$$

= constant in both magnitude and direction at every point of overlap region.

Also E is non zero so potential is not same at all points.

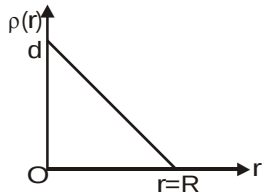
Comprehension Bassed Question

1. $E(4\pi R^2) = \frac{(Ze)}{\epsilon_0} \Rightarrow E = \frac{Ze}{4\pi\,\epsilon_0\,R^2}$

$\Rightarrow$ Independent of a

2. $\because \frac{\rho}{d} + \frac{r}{R} = 1$

$\therefore \rho = \left(1 - \frac{r}{R}\right) d$



Charge contained

$$= Ze = \int_0^R (4\pi r^2 dr)\left(1 - \frac{r}{R}\right) d$$

$$= 4\pi d \int_0^R \left(r^2 - \frac{r^3}{R}\right) dr = 4\pi d\left(\frac{R^3}{3} - \frac{R^4}{4R}\right)$$

$$= 4\pi d\left(\frac{R^2}{12}\right) = \frac{\pi d R^3}{3} \Rightarrow d = \frac{3Ze}{\pi R^3}$$

3. For $E \propto r$, charge density should be constant so a = R

Subjective

1. Capacities of conducting spheres are in the ratio of their radii. Let C_1 and C_2 be the capacities of S_1 and S_2, then $\frac{C_2}{C_1} = \frac{R}{r}$

(i) Charges are distributed in the ratio of their capacities. Let in the first contact, charge acquired by S_2, is q_1,. Therefore, charge on S_1 will be $Q–q_1$.

Say it is q'_1. $\therefore \frac{q_1}{q'_1} = \frac{q_1}{Q - q_1} = \frac{C_2}{C_1} = \frac{R}{r}$

It implies that Q charge is to be distributed in S_2 and S_1 in the ratio of R/r.

$$\therefore q_1 = Q\left(\frac{R}{R+r}\right) \text{ ...(i)}$$

In the second contact, S_1 again acquires the same charge Q. Therefore, total charge in S_1 and S_2 will be

$$Q + q_1 = Q\left(1 + \frac{R}{R+r}\right)$$

This charge is again distributed in the same ratio. Therefore, charge on S_2 in second contact,

$$q_1 = Q\left(1 + \frac{R}{R+r}\right)\left(\frac{R}{R+r}\right) = Q\left[\frac{R}{R+r} + \left(\frac{R}{R+r}\right)^2\right]$$

Similarly

$$q_3 = Q\left[\frac{R}{R+r} + \left(\frac{R}{R+r}\right)^2 + \left(\frac{R}{R+r}\right)^3\right]$$

$$\text{and } q_n = Q\left[\frac{R}{R+r} + \left(\frac{R}{R+r}\right)^2 + \ldots + \left(\frac{R}{R+r}\right)^n\right]$$

$$\Rightarrow q_n = Q\frac{R}{r}\left[1 - \left(\frac{R}{R+r}\right)^n\right] \text{...(i)} \left[S_n = \frac{a(1-r^n)}{(1-r)}\right]$$

Therefore, electrostatic energy of S_2 after n such contacts

$$U_n = \frac{q_n^2}{2C} = \frac{q_n^2}{2(4\pi\varepsilon_0 R)} \Rightarrow U_n = \frac{q_n^2}{8\pi\varepsilon_0 R}$$

where q_n can be written from Eq. (2)

$$\text{(ii) } q_n = \frac{QR}{R+r}\left[1 + \frac{R}{R+r} + \ldots + \ldots + \left(\frac{R}{R+r}\right)^{n-1}\right]$$

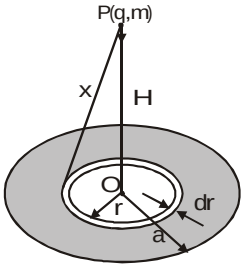
as $n \to \infty$

$$q_\infty = \frac{QR}{R+r}\left(\frac{1}{1-\frac{R}{R+r}}\right) = \frac{QR}{R+r}\left(\frac{R+r}{r}\right) = Q\frac{R}{r}$$

$$\left[S_\infty = \frac{a}{1-r}\right]$$

$$\therefore U_\infty = \frac{q_\infty^2}{2C} = \frac{Q^2R^2/r^2}{8\pi\varepsilon_0 R} \Rightarrow U_\infty = \frac{Q^2R}{8\pi\varepsilon_0 r^2}$$

2. Potential at a height H on the axis of the disc V(P): The charge dq contained in the ring shown in figure Potential of p due to this ring



$$dV = \frac{1}{4\pi\varepsilon_0}\cdot\frac{dq}{x} \text{ where } x = \sqrt{H^2+r^2}$$

$$dV = \frac{1}{4\pi\varepsilon_0}\cdot\frac{(2\pi r dr)\sigma}{\sqrt{H^2+r^2}} = \left(\frac{\sigma}{2\varepsilon_0}\right)\times\left(\frac{rdr}{\sqrt{H^2+r^2}}\right)$$

$\therefore$ Potential due to the complete disc

$$V_p = \int_{r=0}^{r=a} dV = \frac{\sigma}{2\varepsilon_0}\int_{r=0}^{r=a}\frac{rdr}{\sqrt{H^2+r^2}}$$

$$V_p = \frac{\sigma}{2\varepsilon_0}\left[\sqrt{a^2+H^2}-H\right]$$

Potential at centre,(O) will be $V_O = \frac{\sigma a}{2\varepsilon_0}$ $\therefore H=0$

(i) Particle is released from P and it just reaches point O. Therefore, from conservation of mechanical energy.
Decrease in gravitational potential energy = Increase in electrostatic potential energy ($\Delta KE=0$ because $K_1=K_f=0$)

$$\therefore mgh = q[V_O - V_p] \Rightarrow gH = \left(\frac{q}{m}\right)\left(\frac{\sigma}{2\varepsilon_0}\right)$$

$$\left[a-\sqrt{a^2+H^2}+H\right] \text{...(i)}$$

$$\frac{q}{m} = \frac{4\varepsilon_0 g}{\sigma} \quad \therefore \frac{q\sigma}{2\varepsilon_0 m} = 2g$$

Substituting in Eq. (i), we get

$$gH = 2g\left[a+H-\sqrt{a^2+H^2}\right]$$

$$\Rightarrow \frac{H}{2} = (a+H)-\sqrt{a^2+H^2}$$

$$\Rightarrow \sqrt{a^2+H^2} = a+\frac{H}{2}$$

$$\Rightarrow a^2+H^2 = a^2+\frac{H^2}{4}+aH \Rightarrow \frac{3}{4}H^2 = aH$$

$$\Rightarrow H = \frac{4}{3}a \text{ and } H=0 \therefore H = \left(\frac{4}{3}\right)a$$

(ii) Potential energy of the particle at height H = Electrostatic potential energy + gravitational potential energy

$\therefore U = qV + mgH$

Here V= Potential at height H

$$U = \frac{\sigma q}{2\varepsilon_0}\left[\sqrt{a^2+H^2}-H\right]+mgH \text{ ...(ii)}$$

At equilibrium position $F = \frac{-dU}{dH} = 0$

Differentiating Eq. (ii) w.r.t. H

$$\Rightarrow mg + \sigma\frac{q}{2\varepsilon_0}\left[\left(\frac{1}{2}\right)(2H)\frac{1}{\sqrt{a^2+H^2}}-1\right] = 0$$

$$\left(\because \frac{\sigma q}{2\varepsilon_0} = 2mg\right)$$

$$\therefore mg + 2mg\left[\frac{H}{\sqrt{a^2+H^2}}-1\right] = 0$$

$$\Rightarrow 1+\frac{2H}{\sqrt{a^2+H^2}}-2 = 0 \Rightarrow \frac{2H}{\sqrt{a^2+H^2}} = 1$$

$$\Rightarrow \frac{H^2}{a^2+H^2} = \frac{1}{4} \Rightarrow 3H^2 = a^2 \Rightarrow H = \frac{a}{\sqrt{3}}$$

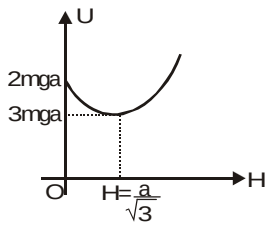
From Eq. (ii), we can write U–H equation as

$$U = mg\left(2\sqrt{a^2+H^2}-H\right)$$

(Parabolic variation) U=2mga at H=0

and ($U=U_{min} = \sqrt{3}mga$ at $H=\frac{a}{\sqrt{3}}$

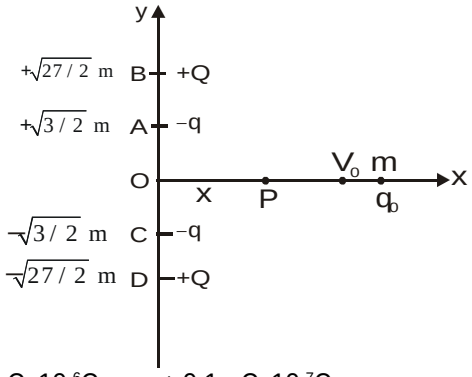
therefore, U–H graph will be as shown.



Note that at $H=\frac{a}{\sqrt{3}}$, U is minimum.

Therefore, $H=\frac{a}{\sqrt{3}}$ is stable equilibrium position.

3. In the figure



$q = 1\mu C = 10^{-6}C$, $q_0 = +0.1\ \mu C = 10^{-7}C$
and
$m = 6 \times 10^{-4}kg$ and $Q = 8\mu C = 8 \times 10^{-6}C$
Let P be any point at a distance x from origin O. Then

$$AP = CP = \sqrt{\frac{3}{2}+x^2}$$

$$BP = DP = \sqrt{\frac{27}{2}+x^2}$$

Electric potential at point P will be

$$V = \frac{2KQ}{BP} - \frac{2Kq}{AP}$$

where $K = \frac{1}{4\pi\varepsilon_0} = 9 \times 10^9 N/m^2/C^2$

$$\therefore V = 2 \times 9 \times 10^9 \left[\frac{8 \times 10^{-6}}{\sqrt{\frac{27}{2}+x^2}} - \frac{10^{-6}}{\sqrt{\frac{3}{2}+x^2}}\right]$$

$$V = 1.8 \times 10^4 \left[\frac{8}{\sqrt{\frac{27}{2}+x^2}} - \frac{1}{\sqrt{\frac{3}{2}+x^2}}\right] ..(i)$$

$\therefore$ Electric field at P is

$$E = -\frac{dV}{dx} = 1.8 \times 10^4$$

$$\left[(8)\left(\frac{-1}{2}\right)\left(\frac{27}{2}+x^2\right)^{-\frac{3}{2}} - \left(-\frac{1}{2}\right)\left(\frac{3}{2}+x^2\right)^{-\frac{3}{2}}\right](2x)$$

E=0 on x–axis where –x = 0 or

$$\frac{8}{\left(\frac{27}{2}+x^2\right)^{3/2}} = \frac{1}{\left(\frac{3}{2}+x^2\right)^{3/2}}$$

$$\Rightarrow \frac{(4)^{3/2}}{\left(\frac{27}{2}+x^2\right)^{3/2}} = \frac{1}{\left(\frac{3}{2}+x^2\right)^{3/2}}$$

$$\Rightarrow \left(\frac{27}{2}+x^2\right) = 4\left(\frac{3}{2}+x^2\right)$$

This equation gives $x = \pm\sqrt{\frac{5}{2}}$ m

The least value of kinetic energyof the particle at infinity should be enough to take the particle upto $x = +\sqrt{\frac{5}{2}}$ m because at $x = +\sqrt{\frac{5}{2}}$ m, E=0

$\Rightarrow$ Electrostatic force on charge q is zero or F_e=0.

For at $x > \sqrt{\frac{5}{2}}$ m, E is repulsive (towards x–axis) and for $x < \sqrt{\frac{5}{2}}$ m, E is attractive (towards negative x–axis).

Now, from Eq. (i), potential at $x = \sqrt{\frac{5}{2}}$ m

$$V = 1.8 \times 10^4 \left[\frac{8}{\sqrt{\frac{27}{2}+\frac{5}{2}}} - \frac{1}{\sqrt{\frac{3}{2}+\frac{5}{2}}}\right]$$

$\Rightarrow V = 2.7 \times 10^4$ volt

Applying energy conservation at $x=\infty$ and $x=\sqrt{\frac{5}{2}}$ m; $\frac{1}{2}mv_0^2 = q_0V$ ...(ii)

$$\frac{1}{2}mv_0^2 = q_0V \text{ ...(ii)}$$

$$\therefore v_0 = \sqrt{\frac{2q_0V}{m}}$$

Substituting the values

$$v_0 = \sqrt{\frac{2\times10^{-7}\times2.7\times10^{4}}{6\times10^{-4}}} \Rightarrow v_0 = 3\ m/s$$

$\therefore$ Minimum value of v_0 is 3m/s

From eq. (i), potential at origin (x=0) is

$$V_0 = 1.8\times10^4\left[\frac{8}{\sqrt{\frac{27}{2}}}-\frac{1}{\sqrt{\frac{3}{2}}}\right]\approx 2.4\times10^4\ V$$

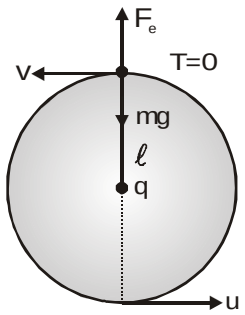
Let K be the kinetic energy of the particle at origin. Applying energy conservation at x=0 and x=∞

$K + q_0V_0 = \frac{1}{2}mv_0^2$ But $\frac{1}{2}mv_0^2 = q_0V$ [from Eq. (ii)]

$K=q_0(V-V_0) = (10^{-7})(2.7\times10^4-2.4\times10^4)$

$\Rightarrow K=3\times10^{-4}$ J

4. Given $q=1\mu C = 10^{-6}C$



$m = 2\times10^{-3}$ kg and $\ell = 0.8$ m

Let u be the speed of the particle at its lowest point and v its speed at high point. At highest point three forces are acting on the particle.

(i) Electrostatic repulsion

$$F_e - \frac{1}{4\pi\varepsilon_0} = \frac{q^2}{\ell^2}\ \text{(outwards)}$$

(ii) Weight W=mg (inwards)

(iii) Tension T (inwards)

T=0, if the particle has just to complete the circle and the necessary centripetal force provided by

$$W-F_e \text{ i.e. } \frac{mv^2}{\ell} = W - F_e$$

$$\Rightarrow v^2 = \frac{\ell}{m}\left(mg - \frac{1}{4\pi\varepsilon_0}\times\frac{q^2}{\ell^2}\right)$$

$$v^2 = \frac{0.8}{2\times10^3}\left(2\times10^3\times10 - \frac{9.0\times10^9\times(10^{-6})^2}{(0.8)^2}\right)$$

$\Rightarrow v^2 = 2.4\ m^2/s^2$ ...(i)

Now, the electrostatic potential energy at the lowest and highest points are equal. Hence, from conservation of mechanical energy.

Increase in gravitational potential energy

= Decrease in kinetic energy.

$$\Rightarrow mg(2\ell) = \frac{1}{2}m(u^2 - v^2) \Rightarrow u^2-v^2=4g\ell$$

Substituting the values of v^2 from Eq. (i), we get

$u^2 = 2.4 + 4(10)(0.8) = 34.4\ m^2/s^2$

$\therefore u = 5.86$ m/s

Therefore, minimum horizontal velocity imparted to the lower ball, so that it can make complete revolution, is 5.86 m/s.

5. For potential energy of the system of charges, total number of charge pairs will be $^8C_2$ or 28 of these 28 pairs 12 unlike charges are at a separation a, 12 like charges are at separation $\sqrt{2}$ a and 4 unlike charges are at separation $\sqrt{3}$ a. Therefore, the potential energy of the system

$$U=\frac{1}{4\pi\varepsilon_0}\left[\frac{(12)(q)(-q)}{a}+\frac{(12)(q)(q)}{\sqrt{2}a}+\frac{(4)(q)(-q)}{\sqrt{3}a}\right]$$

$$=-5.824\left(\frac{1}{4\pi\varepsilon_0}\cdot\frac{q^2}{a}\right)$$

The binding energy of this system is therefore,

6. (i) Applying energy conservation principle, increase in kinetic energy of the dipole=decrease in electrostatic potential energy of the dipole.

$$\Rightarrow KE=0-\left(-\vec{p}\cdot\vec{E}\right) = \vec{p}\cdot\vec{E} = (p\hat{i})\cdot\left(\frac{1}{4\pi\varepsilon_0}\frac{q}{d^2}\hat{i}\right) = \frac{qp}{4\pi\varepsilon_0 d^2}$$

(ii) Electric field at origin due to the dipole,

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{2p}{d^3}\hat{i}\left(\vec{E}_{axis}\uparrow\uparrow\vec{p}\right)$$

$\therefore$ Force on charge q at distance d

$$\Rightarrow \vec{F} = q\vec{E} = \frac{pq}{2\pi\varepsilon_0 d^3}\hat{i}$$

7. Electric field near a large metallic plate is given by

$E=\sigma/\varepsilon_0$. In between the plates the two fields will be in opposite direction. Hence,

$$E_{net}=\frac{\sigma_1-\sigma_2}{\varepsilon_0}=E_0 \text{ (say)}$$

Now, W=(q) (potential difference) =q (E_0acos45°)

W=(q) =q (E_0acos45°)

$$=(q)\left(\frac{\sigma_1-\sigma_2}{\varepsilon_0}\right)\left(\frac{a}{\sqrt{2}}\right)=\frac{(\sigma_1-\sigma_2)qa}{\sqrt{2}\varepsilon_0}$$

8. Let q be the charge on the bubble, then

$$V=\frac{Kq}{a}\left(\text{Here } K=\frac{1}{4\pi\varepsilon_0}\right) \quad \therefore \quad q=\frac{Va}{K}$$

Let after collapsing the radius of droplet becomes R, then equating the volume, we have

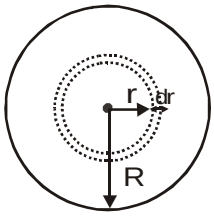
$$(4\pi a^2)t=\frac{4}{3}\pi R^3 \qquad \therefore R=(3a^2t)^{1/3}$$

Now, potential of droplet will be $V'=\frac{Kq}{R}$

Substituting the values, we have

$$V'=\frac{(K)\left(\frac{Va}{K}\right)}{(3a^2t)^{1/3}} \Rightarrow V'=V\left(\frac{a}{3t}\right)^{1/3}$$

9. $$E\left(4\pi r^2\right)=\int\frac{\left(kr^a\right)4\pi r^2dr}{\epsilon_0}$$



$$E(r^2)=\frac{\int r^{a+2}dr}{\epsilon_0}$$

$$\Rightarrow E(r^2)=\frac{r^{a+3}}{(a+3)\epsilon_0} \Rightarrow E=\frac{r^{a+1}}{(a+3)\epsilon_0}$$

$$\Rightarrow E(R/2)=E\frac{1}{8}E(R) \Rightarrow \left(\frac{R}{2}\right)^{a+1}=\frac{1}{8}(R)$$

$$\Rightarrow 2^{a+1}=2^3 \Rightarrow a=2$$